A DIGITAL PROGRAM FOR TRANSIENT CALCULATIONS IN ELECTRICAL NETWORKS

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CERTIFICATE

This dissertation has not been, nor is being currently submitted for the award of any other degree or similar qualification.

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1. INTRODUCTION

1.1. The Transient Problem

Electrical Engineering is an applied science covering a very wide variety of specialised fields. This specialisation is becoming ever more pronounced until it often appears that one branch of the subject has very little in common with any other.

It may, of course, be argued that there are certain 'core' subjects which are common to all branches of the science - network analysis could be cited as an example. Even here, however, diversity is becoming the order of the day. In power engineering, network analysis is now mainly characterised by load flow, symmetrical and unsymmetrical fault calculations, and transient stability analyses. None of these, however, has much direct application to the analysis of electronic circuits.

It is interesting, therefore, to meet a subject which has a wide general application to a range of heavy and light current situations. Such a subject is the study of transient phenomena since it may be shown that the same general methods of approach are equally viable - and what is more important produce useful practical results - for a whole range of electrical engineering problems.

Interest in transient phenomena has for some years been centered on heavy current or power engineering. It is here that the most 'spectacular' transient events are found - witness the effects of lighting on a power distribution network, or the operation of a circuit breaker in a high voltage supply network. The techniques which form the basis of this work were first applied to the solution of power system problems, especially for the analysis - in most cases using the digital computer - of situations involving the generation of overvoltages within the network under examination.

The analysis of electronic circuits and of communication networks, on the other hand, has traditionally been largely confined to the frequency domain. The rapid rise of digital electronics over the past few years has however lead to a situation where time domain solutions are becoming important.

1.2. Lumped And Distributed Constants

The comments made above may have implied that classical network analysis - especially that applied to light current problems - includes no mention of transient phenomena. Any reference to syllabi for a course in Electrical Engineering will show at once that this is not the case. It is necessary, however, to make the distinction between classical 'transient phenomena' which for the most part deals with transients in networks containing only lumped elements, and the more general type of study which will be undertaken here and which provides that networks may contain both distributed constant and lumped constant branches.

The division between what is a lumped element and what is one with distributed constants is superficially well defined. It is normal to consider that an element having only a small electrical length - for example a resistor, inductor or capacitor - falls into the former category, whilst the classic example of a distributed constant branch is the transmission line.

In most cases this simple division works well though it is sometimes necessary to excercise discretion, usually based on experience, in particular cases. An example is a power transformer. If the network being examined consists, say, of a long overhead transmission line, perhaps having a length approaching

100km, with a corresponding transmission of 'transit' time of some 0.33 microseconds, then it might prove reasonable to consider the transformer as being represented by an inductor or even more simply - and possibly more accurately - as a simple resistor of high value. In this case, therefore, the transformer is being simplified and treated as a lumped element.

On the other hand the transformer itself could form the main topic of interest. Typically the analyst would be interested in determining the distribution of voltage along the windings under various terminal voltage conditions. Here, then, some form of distributed constant, or at least lumped element approximation to a distributed constant network would be required to represent the transformer fully. Fig. 1.1. shows typical transformer representations as (a) a lumped element, and (b) a lumped element approximation to a distributed constant system. This latter circuit is due to Bewley[1].

It is well known that it is mathematically feasible to represent a distributed constant system by a lumped element 'equivalent circuit' - this was implied in case (a) for the transformer mentioned above. Indeed this idea is of dominant importance in several of the techniques used for transient analysis which will be discussed in a subsequent chapter. Such a representation, however, implies an approximation which will be zeroed only if an infinite number of lumped elements is used. In practical situations, however, this 'equivalent circuit' technique proves to be a viable idea capable of producing useful results. It is not so often appreciated that the reverse operation - namely of representing a lumped reactive element, an inductor or a capacitor, by a transmission line - is also possible though here again there is an implied approximation which may, however, be kept small. This idea would appear to have little practical significance since classical theory is capable of handling pure lumped

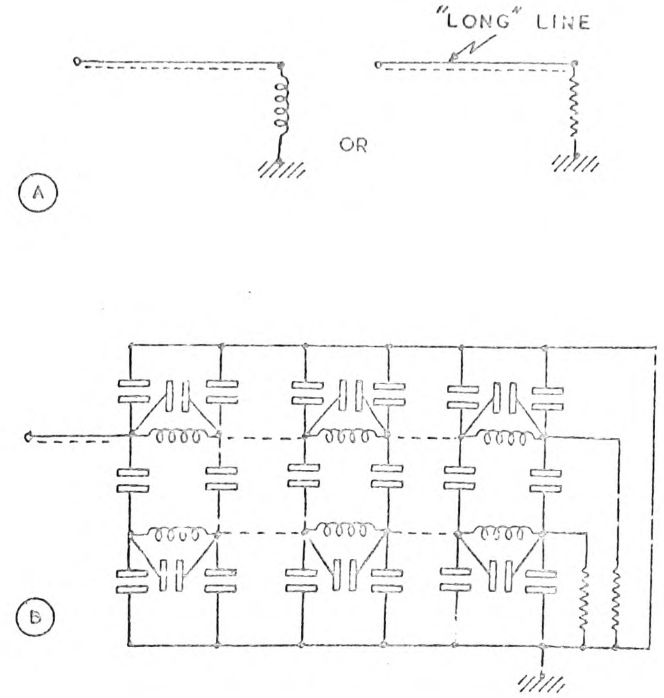


FIG. 1.1.

elements more readily than it is of handling, say, a transmission line. However using the technique - the 'graphical method' - which forms the central feature of this work, it does, in fact, prove simpler to analyse networks consisting predominantly of distributed constant branches than those of the more traditional kind. In this case, therefore, this idea of equivalencing a lumped reactive element to a transmission line assumes some importance. The idea itself will, of course, be familiar to students of communication theory where the reactive nature of transmission lines whose length corresponds to certain multiples of a wavelength at the operating frequency is well known.

1.3. Transients in Power Systems

Though the growing importance of transient studies in the power system field has already been mentioned, it is convenient here to examine quickly in more detail the necessity for studies of this kind.

Until recent years, and particularly until the development of the general purpose digital computer, interest in power system transient phenomena has been mainly directed towards examination of the observable effects of such transients on system performance. Much work has been carried out - some of it statistical in nature - on the frequency, severity and effects of lightning and other disturbances on typical power supply networks.

Comparatively little, with some notable exceptions such as the work of Heller et al[2] and Rudenberg[3], has been done on the other hand of a purely analytic type. By this is meant the computation of the time (or Frequency) response of a power supply network to given forcing functions. There are a number of reasons for this lack of attention, some of which may be summarised as follows:-

1.3.1. Lack of computing facilities.

Apart from studies on simple systems where some form of graphical technique can be employed, or where the time and effort of manual computation can be justified, most practical analyses require the use of a digital computer. Attempts to predict transients using analog techniques by the use of model systems and by the development of the more sophisticated Transient Network Analyser (TNA) have met with some success[4]. The ~~facts~~ fate of such analysers, however, is destined to be that of the more conventional A.C. networks analysers which are tending to obsolescence with the development of ever more sophisticated digital computer programs.

1.3.2. The nebulous nature of the lighting mechanism.

Lightning strikes, and to a lesser extent switching operations are relatively unpredictable occurrences. They are unpredictable not only as far as timing is concerned, but also in such other vital factors as peak voltage, waveshape, source impedance and so on. It may thus be argued that any attempt to predict the performance of a network - in the sense of computing voltage and current levels as functions of time - following a lightning strike or switching operation can at best be an educated guess since so many assumptions must be made.

1.3.3. The lack of background work in the field.

In particular the scarcity of generally accepted mathematical models for a power network under transient conditions.

Even now the amount of information available as input data for a typical digital transient program is limited mainly because relevant measurements on items of plant have never been taken. It is, for example, very difficult to obtain information on, say, the terminal capacitance to ground of a power transformer since it is most unlikely that this data has ever been recorded. Data for steady state studies, on the other hand, is normally readily available and well defined.

The increasing level of power system operating voltage has increased the dangers of damage due to surges caused by switching operations. Indeed the only recorded damage in the form of flashovers or blowouts on the British supergrid during the past few years has been ascribed to switching surges.

Additionally the rising capital costs of protective equipment - such as surge divertors - for the new very high voltage systems has meant that the old 'brute force' approach to insulation co-ordination wherein divertors would be placed at every seemingly logical point of risk has had to be reconsidered. All of these factors contribute to the necessity for system transient analysis despite the problems already outlined.

The fact that switching surges, whose forcing functions are largely predictable, now constitute the biggest problem eliminates to some extent the objections of 1.3.2. to time domain transient analysis. Even if we are concerned with lighting, however, there is much useful work which, in the writers opinion, can still be done. The policy should be to determine trends and establish principles of practice rather than to attempt to analyse specific situations. It may be readily demonstrated that some empirically designed protection schemes widely used throughout the industry are of little practical use and even in some cases worsen the situation which they hope to alleviate. An example might be the siting of a length of cable between the overhead line and the equipment to be protected - say a transformer - as shown in fig. 1.2. This operation has often been supposed to afford protection to the transformer which in some circumstances it manifestly fails to do.

1.4. Transient Calculations for Electronic Systems

Fundamentally the same problems which exist in the heavy current sphere are carried over into the domain of electronic circuitry and particularly microelectronics. The differences are mainly ones of scale especially time, voltage and current. Instead of dealing with transmission lines whose lengths are measured in kilometres, we are now concerned with fractions of metres and corresponding transit times measured most conveniently in nanoseconds.

The digital computer, the primary tool for this fast transient analysis, itself provides the problems to be analysed. Interest now centres on the distortions which are produced in pulses as a result of their transmission through the system. Additionally the overvoltage problem can still be important especially in systems using integrated circuits with their rather limited tolerance to transients of this kind. The obvious solution - a panacea for all transient problems - is, of course, to match as nearly as possible each system element to the elements driven by it and driving it. Such a simple solution proves impractical for a variety of reasons. Not least amongst these is that such a practice leads to optimal power transfer levels and consequently to a much higher power dissipation and temperature rise than is tolerable in modern high density electronic assemblies.

FIG. 1.2.

Modern tendancies are, therefore, to use high input and low output impedances which produce almost ideal conditions for the generation of overvoltages and other transient effects.

The growing need to perform time domain analyses in electronic systems having distributed constant elements has been reflected in the numbers of papers starting to appear on this subject.[5,6,7]

1.5. A Statement of Objectives

Given that the problem at hand is to be able to compute fast transients in a variety of heavy and light current systems, the primary object of this work has been to produce a program having as high a degree of sophistication as possible to perform such calculations and present the results in a useful form.

The work 'sophistication' in this statement require some amplification. The program specification - which tended to evolve rather than be specifically designed - had certain major features. These can be summarised as follows :-

1.5.1.

The program had to be 'general purpose' in the sense that it could handle a variety of transient problems with no re-programming being necessary on the part of the user. In a sense, therefore, it had to be developed to the stage of being a production program rather than merely existing as an equation solving routine.

1.5.2.

The system had to be easy to use with a minimum of instruction. This necessitated the inclusion of quite complex error detection and message routines. It is so often the case, as here, that the simpler a program of this kind is made to use - in the sense of ease of data preparation etc. - the more complicated is the programming effort involved.

1.5.3.

The program output had to be as flexible as possible, provision being made for hard copy, graphical output etc., at the discretion of the user. A feature of the system is the manual interaction facilities which it offers and which are of particular usefulness when it is operated on a small computer such as the I.B.M.1130 on which the majority of the work was carried out.

The resulting system - which in the current fashion has been christened SUSAN (SUrge System ANalysis program) - is written in Fortran and an idea of its complexity can be gained from its size which extends to 2500 cards of source program. An early version of SUSAN has been deposited in the Institution of Electrical Engineers computer program library.[8] A portion of this work is thus naturally concerned with a description of the program with special reference to some of the features which are of the most interest.

A program can be judged only on the results which it produces and on the efficiency of their production. Accordingly a number of analyses are considered. Some of these are included simply as a means of demonstrating the capabilities of SUSAN whilst others, besides serving this purpose, have additional significance since they allow some general conclusions to be drawn about the transient behaviour of networks. Examples, chosen to be as practical as possible, have been selected from both the heavy and light current fields of interest. In addition, in an attempt to obtain correlation between computed and practical results a certain amount of laboratory work using co-axial cables and other components to represent the system under examination has been carried out.

The availability of modern sampling oscilloscopes and, in particular, the time domain reflectometer makes measurements possible in system of this kind.

Prior to a discussion of the program itself and its theoretical background, it is considered apropriate in the following chapter to conduct a brief survey of alternative techniques which have been employed for this type of transient analysis.

2. TRANSIENT CALCULATION TECHNIQUES

2.1. Methods of Approach

There now exist a number of techniques for transient computation, many of which have been adapted to suit the needs of the digital computer in the past few years. To date a 'best method' for work of this kind has not emerged - at least by general concensus. The situation is similar to that which surrounded the 'load-flow' problem in power system steady state analysis some ten years ago. Nevertheless several of the procedures, especially in recent times, have gained ground at the expense of the others. The program forming the basis of this work employs a variation on the 'Bergeron' or 'graphical' method. Whilst the technique proves to be adequate for a whole range of problems, the writer would certainly not claim that it is the 'best' nor indeed necessarily better than competitive systems. It may well prove that the procedure which eventually gains widespread acceptance will be an amalgam or hybrid between other techniques.

The existing methods may be broadly divided under several headings. The traditional method of calculating transients, especially in power systems, has been to use the transient analyser - in reality a specialist analog computer. Here distributed constant lines and cables are represented by artificial lines made up of lumped elements and arranged, normally, in a series of 'pi' sections. One of the earliest digital techniques sought simply to duplicate this process on the digital computer thus giving rise to the so-called 'lumped parameter' method.

All of the remaining techniques acknowledge the distributed nature of transmission lines. One of the oldest practical procedures utilises the D'Alembert solutions of the wave equations for voltages and current on such lines. As is well known these solutions involve the concepts of waves travelling along the lines. This procedure is, of course, the 'lattice method' usually attributed to Bewley[2]. The same D'Alembert solutions also form the basis of the 'graphical method' though this differs in basic philosophy from the lattice method and must be considered in isolation from it.

The lattice and graphical methods, existing as they do entirely in the time domain, suffer from the disadvantage that the frequency dependence of transmission line and other parameters cannot be taken into account. Additionally line attenuation is more difficult to simulate, especially in the case of the graphical method. This disadvantage is probably not so apparent in practice as might be supposed though in an attempt to overcome it various frequency domain techniques have been proposed. Prominent amongst these are methods based on the use of the Fourier Transform, especially in a 'modified' form where a 'sigma factor' is included in an attempt to damp Gibb's oscillation which is otherwise a problem. Use of the Laplace transform has also been proposed though this would appear to offer few advantages over other methods.

In addition various alternative techniques are proposed from time to time though analysis usually shows that they are, in fact, variations on one or other of the basic methods mentioned above, or are systems composed of combinations of these methods. For example a combination of the lattice method with a Fourier Transform technique would appear to hold some promise.

2.2 'Lumped Parameter' technique

One of the most important practical applications of this procedure to date has been in its use for the calculation of circuit breaker restriking transients and here the work of Bickford[9] must be considered important. An example will serve to illustrate the technique.

Fig. 2.1 shows a simple network consisting of a transmission line, Aß, feeding a second line through a three-phase circuit breaker at ß. A three-phase fault has occurred on the breaker terminals as shown. The breaker opens to clear the fault, but it is most unlikely that all three poles of the breaker will open together. Accordingly a heavy restriking transient will be produced across the first pole to clear.

The transmission line (each phase) is represented by one or more 'pi' equivalent circuits much in the same manner as a medium length line would be in the case of steady state A.C. analysis. The alternator may be represented by its inductance per phase. The effective equivalent circuit is thus that shown in fig. 2.2, which may be simplified to that shown in fig. 2.3.

To simulate the restriking transient, a current injection is made into the breaker terminals (x-y). The value of this current is equal and opposite to the current which would have flowed had the breaker pole remained closed. The remainder of the procedure is similar to conventional nodal circuit analysis with the exception that the resulting simultaneous equations contain derivative terms. These equations are normally solved using an integration routine of the Ruge-Kutta type. The complexity of the solution depends on the number of lumped elements in the final network and also on the nature of the forcing function. For most power system applications, sinusoidal forcing functions will be required though if the time of interest is short a step or ramp approximation

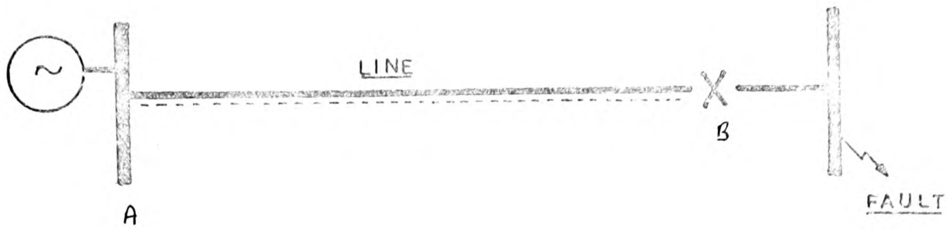


FIG. 2.1.

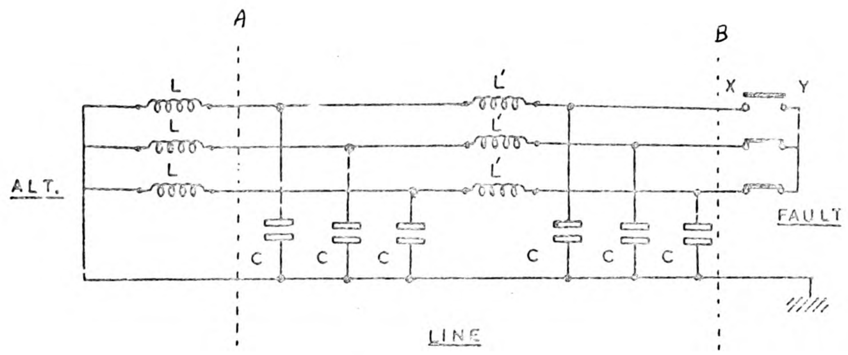


FIG. 2.2.

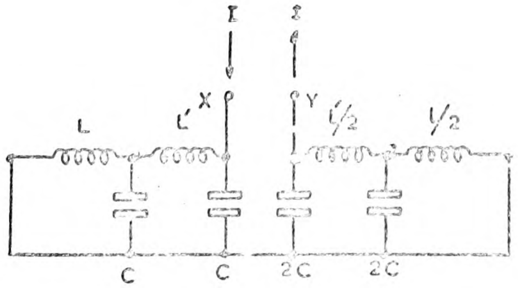


FIG. 2.3.

to a sinusoid is often used.

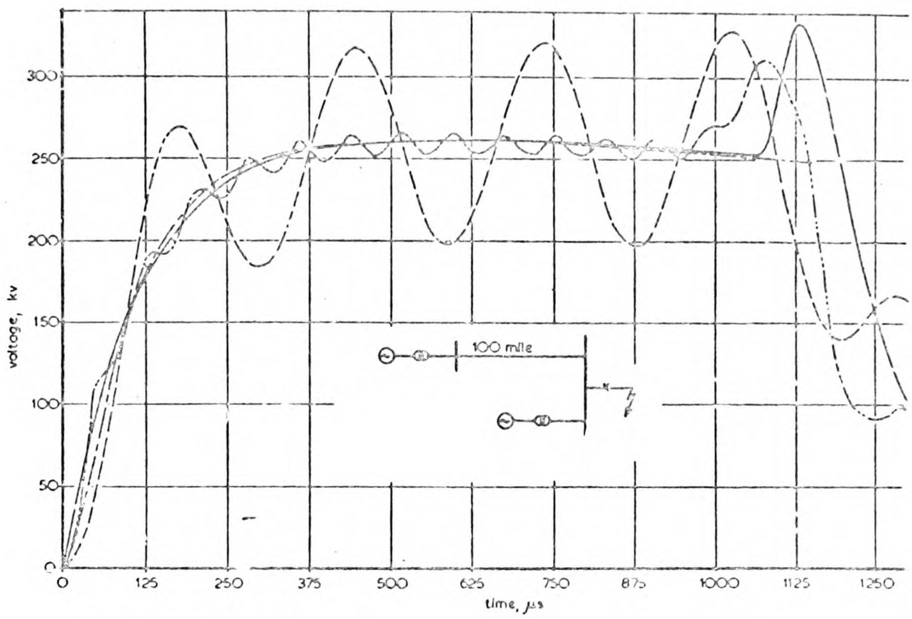
2.2.1 Accuracy of Transmission Line Representation

In the simple illustrative example given above the distributed constant transmission line was represented by a single 'pi' section. In practice as many as twenty such sections can be used for a long line. In general the more sections that are used, the more accurately does the computed response approach the practical result, however the more complex is the solution and consequently the greater is the amount of computer time required.

Fig. 2.4. (after Bickford[10]) illustrates two typical restriking voltage calculations involving a line 138 km. long being represented by 5 and 20 'pi' sections respectively. The pronounced low frequency oscillation in the former case is not a characteristic of the system under investigation, but is produced as a consequence of the 'coarse' representation of the transmission line.

This type of transmission line representation may be considered accurate at a single frequency or over a limited range of frequencies. The ladder network, however, constitutes a filter whose bandwidth approximates to that of the natural frequency of one of the 'pi' sections. Obviously the greater number of sections used then the smaller will be the component values associated with each section and hence an increase in bandwidth - and thus accuracy in the time domain - will result.

Bickford has argued, correctly in the writer's opinion, that this procedure is useful for restriking transients or for other situations where the forcing function does not contain high



Effect of line representation

— — — — lumped-parameter method, line represented by five π sections
—··—··— lumped-parameter method, line represented by 20 π sections
—·—·—· lumped-parameter method, line represented by surge impedance
———— lattice-diagram solution

FIG. 24

frequency components. This implies that applications requiring, say, step function excitation are not particularly suited to this treatment.

The lumped parameter technique has now been largely abandoned in favour of alternative procedures, not only on account of its accuracy limitations, but also because it is relatively uneconomic in computer time and storage requirements and because it is capable of handling systems containing non-linear elements only with great difficulty.

2.3 The 'Lattice Method'

In a sense the lattice method is the best known of all the transient computation techniques and is the one which, until comparatively recently, has found a place in many undergraduate courses. The advantages of this technique from an educational point of view are that it is relatively easy to apply to simple problems, may be given a graphical interpretation, and that it gives a good insight into the mechanisms of transient phenomena. The procedure has been usually attributed to Bewley[1,1] though work on similar lines has been produced by Boehne[11].

The basis of the lattice method derives from the standard D'Alembert solutions of the wave equations for a distributed constant line where line resistance and conductance are neglected.

For such a line we have that :-

$$-\frac{\partial V}{\partial x} = L.\frac{\partial i}{\partial t} \qquad \ldots\ldots\ldots\ldots\ 2.1$$

$$-\frac{\partial i}{\partial x} = C.\frac{\partial v}{\partial t} \qquad \ldots\ldots\ldots\ldots\ 2.2$$

Differentiating these two equations and subsequently substituting from one to the other permits the elimination of one of the two dependent variables from each yielding:-

$$\frac{\partial^2 i}{\partial x^2} = \frac{1}{a^2} \cdot \frac{\partial^2 i}{\partial t^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.3$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{a^2} \cdot \frac{\partial^2 v}{\partial t^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.4$$

There are numbers of possible solutions for equations of this type though, as mentioned above, the solution adopted here is that due to D'Alambert. The solution, which is described in the literature[12] is:-

$$V_{x,t} = Z.F1.(x - at) - Z.F2.(x + at) \quad \ldots\ldots\ldots \quad 2.5$$

$$i_{x,t} = F1.(x - at) + F2.(x + at) \quad \ldots\ldots\ldots \quad 2.6$$

Here F1 and F2 are arbitrary functions defined in general by the transmission line boundary conditions. The two functions (x - at) and (x + at) may be interpreted as waves travelling in the positive and negative reference directions respectively.

A transmission line of surge impedance Z1 may now be considered which is terminated in an impedance Z2 (fig. 2.5). In case (a) a voltage wave, V+, with an associated current wave, i+, are moving in the positive reference direction towards the junction. Fig. 2.5b shows the situation some little time later after the waves have reached the junction and been partially reflected giving rise to the reflected waves V- and i- now moving away from the junction in the negative reference direction.

We have that :-

$$V+ = Z1.i+ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.7$$
$$V- = -Z1.i- \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.8$$

The total voltage at the termination is V where:-

$$V = V+ + V- \quad \ldots\ldots\ldots\ldots\ldots\ldots 2.9$$

Similarly the total current through the terminating impedance is i where:-

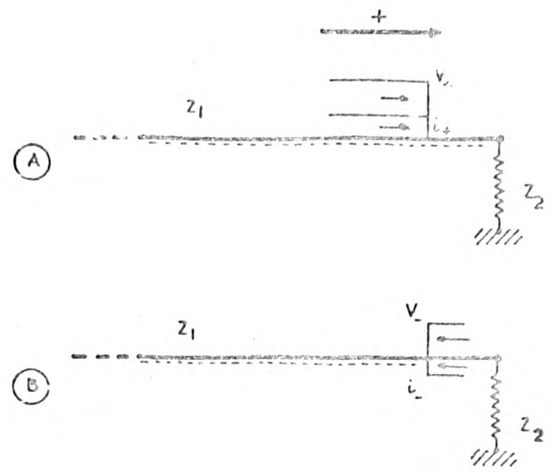

+
V_+
Z_1
i_+
A
Z_2
V_-
Z_1
B
i_-
Z_2

FIG. 25.

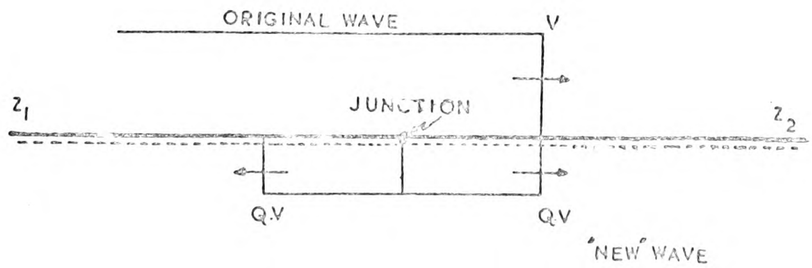

ORIGINAL WAVE
V
Z_1
JUNCTION
Z_2
Q.V
Q.V
"NEW" WAVE

FIG. 2.6.

$$i = i+ + i- \quad .... 2.10$$

Now:-

$$V = v+ + v- = Z2.i = Z2.(i+ + i-) \quad ....2.11$$

Sunstituting from equations 2.7 and 2.8, equation 2.11 becomes:-

$$V = Z2.(v+/Z1 - v-/Z1) = v+ + v- \quad .... 2.11a$$

Therefore:-

$$v- = ((Z2 - Z1)/(Z2 + Z1)).v+ = \varrho.v+ \quad ....2.12$$

The term ϱ in equation 2.10 is the reflection coefficient at the junction. In a similar manner a refraction coefficient (equal to 2.Z2/(Z1 + Z2)) may be defined.

Using these concepts of reflection and refraction coefficients, it is thus possible to compute the effect - in terms of the generation of new travelling waves - of a wave arriving at a junction. The total voltage at any such junction is thus the sum of all of the travelling waves arriving at a junction at a given time.

In practice a slight variant on the basic technique outlined above is adopted in many programs. Here it is assumed that any incoming wave at a junction passes through that junction unchanged, but at the same time generates a new wave having a magnitude of ϱ times the original wave. This new wave proceeds from the junction in both directions as illustrated in fig. 2.6. Superposing all the waves present at a junction - both those produced as a result of the initial disturbance causing the transient and those produced at junctions throughout the network - thus determines the conditions at the junction at any time.

This lattice method has probably been the one most commonly employed in those transient computation routines which acknowledge the distributed nature of transmission lines. It has, at the time of writing, fallen 'out of fashion' to a certain extent, though numbers of workers have described programs based on the technique in the literature [13,14]. These papers describe how the basic technique which has been outlined above may be adapted for a variety of circuit situations. Virtually any form of exciting function may be readily represented and lumped elements may be handled - albeit with some approximation in the case of reactive components.

2.4 Fourier & Related Transform Methods

One of the disadvantages of transient calculation techniques which function in the time domain is their inability to make allowance for the frequency dependence of transmission line parameters. Surprisingly such frequency dependence is of greater importance in studies on power system networks than in the transient analysis of light current arrangements. In the case of the analysis of electronic systems the dominant frequencies will usually be very high due to the 'short' nature of the transmission lines. At such high frequencies it is usually reasonable to assume that an almost perfect 'skin effect' occurs and that there is no internal flux within the line conductors. Additionally most electronic systems have an almost perfect earth - or at least an earth whose parameters are defined better than those of the earth associated with a typical power line which may show wide variations with frequency.

The frequency dependance of the system parameters can be handled by the use of methods based on the Fourier transform. Basically the method requires the calculation of the response of the system over a range of frequencies, and the use of the inverse Fourier transform to transform the response from the frequency to the time domain.

The frequency response of a single phase transmission line is $A(w)$ where:-

$$A(w) = e^{-\gamma(w)x} \qquad \ldots\ldots\ldots\ldots \; 2.13$$

In this equation, x is the line length and γ is the line propagation coefficient. It is well known that:-

$$\gamma(w) = (Zs(w) \cdot Ys(w))^{\frac{1}{2}} \qquad \ldots\ldots\ldots\ldots 2.14$$

and this coefficient may be computed at various frequencies. In the case of power line calculations, the effects of a non-perfect earth may be incorporated into the computation by the use of Carson's formulae[15].

The frequency response of the line to a step input is given by:-

$$A(w) = \frac{e^{-\gamma(w)x}}{jw} \qquad \ldots\ldots\ldots\ldots 2.15$$

and from this the time domain response may be determined by an application of the inverse Fourier transform. We thus have:-

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-\gamma(w)x}}{jw} \cdot e^{jwt} . dw \;\ldots\ldots\; 2.16$$

One of the limitations of this technique centres around the necessity to truncate the infinite range of integration for practical computation. If the infinite integral is not reasonably convergent - as might be the case in many typical situations - the time response is marred by Gibb's oscillation. This phenomena is illustrated in fig. 2.7 (curve 'a') which shows a typical integral representation of a step waveform (after Day et. al.)[16].

Lanczos[17] showed that this type of oscillation could be largely eliminated by the use of his 'sigma factor'. Equation 2.16, modified to include the sigma factor, and with the limits of integration truncated to $-\alpha, \alpha$ becomes:-

$$F(t) = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} \sigma . \frac{e^{-\gamma(\omega)x}}{j\omega} . e^{j\omega t} . d\omega \;\ldots\ldots\; 2.17$$

where:-

$$\sigma = \frac{\sin(\pi\omega/\alpha)}{\pi\omega/\alpha}$$

The effect of such a modification is illustrated in curve 'b' of fig. 2.7 which shows the integral representation of the same step function considered previously. Here the Gibb's oscillation has been almost entirely removed, though at the expense of a lowering of the rate of rise of the function near the origin. Day et. al[16,18] suggested modifications to the sigma factor which further improved the efficiency of the process. They differentiated between the 'standard' sigma factor - that described above - and their modification termed the 'modified sigma factor'.

Battison et al[19] recognise that the standard Fourier transform can yield divergent integrals when applied to some situations. They, therefore, adopt a modified form of the transform where jw terms are replaced by (a + jw) where 'a' is arbitrary and greater than zero. Using this modified transform, coupled with the 'standard' sigma factor they have demonstrated the viability of this technique on relatively simple systems.

2.4.1 Use of the Laplace Transform

Laplace transform techniques have been widely used for many years in the transient analysis of lumped element networks. From time to time the use of this system is proposed for networks having distributed constants. The ability to take into account the frequency dependence of the transmission line parameters is, of course, lost through this method, though it may be considered to have some advantages if the system under examination is relatively simple and contains lumped elements in addition to transmission lines.

One of the most recent applications of the Laplace transform in this context was made by Wassel[20] who was

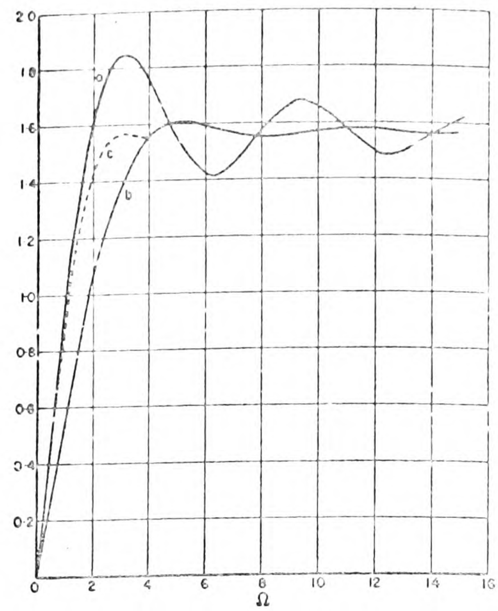


Fig. 3. *Integral representation of step function of magnitude* $\pi/2$

(a) $\mathrm{Si}(\Omega t)$

(b) $\mathrm{Si}_\sigma(\Omega t)$ *with standard sigma factor*

(c) $\mathrm{Si}_\sigma(\Omega t)$ *with modified sigma factor*

FIG.2.7

concerned with reflections produced by the propagation of a step function along a line terminated at each end by a parallel resistor/ capacitor combination. This work will be discussed in more detail at a later time though examinations of Wassel's analysis show that the resulting expressions for the line node voltages as functions of time become extremely complex. The Laplace transform cannot be considered to be a practical approach for any but the simplest of networks.

Transform methods in general can only be used for linear systems. Such techniques are, however, capable of providing results of higher accuracy than that attainable using other processes. In order to achieve such results it is necessary both to provide a great deal of system data - which is often lacking in practical situations - and to make use of considerably more computer storage and time than might be the case using alternative techniques.

It can be argued, therefore, that for studies on large systems, or in situations where the network data is not well defined, or indeed for 'general purpose' studies, these transform methods do not have significant advantages over other simpler and more economic techniques.

2.5 'Hybrid' processes

Bickford and Doepel[10] have suggested a technique which effectively constitutes an amalgam of the lattice diagram approach of Bewley with the Fourier transform technique described above. Here the response of a line to a step function is determined using Fourier methods. The calculation is performed once only for each line in the system. The major portion of the arithmetic then centres around the use of the lattice diagram except that the amplitude of each wave moving along a line is modified according to the step response for that line previously determined.

This composite process thus appears to hold some promises, combining as it does the flexibility and economy of the Bewley method with the improvements in accuracy afforded by the inclusion of line attenuation and frequency dependence.

3. THE "GRAPHICAL" METHOD

3.1 The Background

The computer program forming the basis of this work has been formulated using the principles embodied in the "graphical" method of transient calculation.

This process is more commonly referred to as the "Bergeron" or "Schnyder-Bergeron" method, though the ~~destination~~ designation "graphical" was first assigned to the technique by Bergeron himself.

Most workers would agree that the credit for the conception of the technique should do to Louis Bergeron. His book on the subject[21], published originally in 1931 and subsequently translated into English in 1949, gives a concise description of the method though the majority of applications refer to problems in hydraulics, especially to the "water hammer" problem. O. Schnyder[22], working independently of Bergeron and slightly preceeding him, developed a similar though rather less conclusive technique.

Whilst the method appears to have remained 'popular' in the Civil Engineering field - though here many recent writers, for example Rich[23], refer not to the Bergeron method but to the related Allievi[24] and Angus method - this popularity has not, until quite lately, extended to Electrical Engineering. Interest in the electrical applications of the process waned somewhat during the period between the work of Dojuhass[25] in 1939 and the 1950's when it was 'rediscovered' particularly by Boehne[26].

One of the special virtues of the graphical method is that it provides an excellent tool for teaching. The writer and his colleagues have developed the technique to the point where it has found a useful place in several degree and related courses in Electrical Engineering. A number of publications have resulted from this aspect of the work [27,28,29]. These describe the basic principles of the method - in its purely graphical form where a digital computer is not employed - and also give applications of the technique to the solution of power system transient problems. A description of an adaptation of the process to enable it to handle networks containing non-linear reactive elements is given in ref. 29.

As its name implies the graphical method - as originally formulated - requires the use of drawing to produce results. The complexity of networks which can be handled, and the accuracy of the solutions obtained, is naturally limited by the manual processes involved. It is not surprising, therefore, that the digital computer should be employed to automate the technique and thus turn a useful procedure into an extremely powerful tool.

One of the first recorded digital programs employing the graphical method was for the calculation of water hammer in hydraulic plants[30]. Two years later, in 1961, Frey and Althammer[31] described a routine capable of transient calculation in simple distributed constant electrical networks. This work was further extended by Althammer in 1963[32]. Relatively simple programs illustrating how the technique could be adapted for the digital computer were described by the writer and his colleagues at various conferences and elsewhere during the period 1965 to 1967.[33,34,35] Interest during those years was mainly concerned with power systems transients, particularly as a result of problems which were becoming apparent in various places following the introduction of very high voltage cable links into grid systems.

3.2. The Choice of the Graphical Method

From the preceeding brief survey of the major techniques employed for transient calculation, it is clear that each possesses advantages - and disadvantages - relative to each of the others.

It was stated that, in the writer's opinion, a 'best method' had not emerged and the hybrid combinations of several techniques would seem to offer promise, nevertheless the graphical method has a number of specific attributes which render it useful for general-purpose work. These may be listed as follows :-

(i) The system provides the ability to analyse circuits containing both lumped and distributed elements to a degree of accuracy sufficient for most engineering applications.

(ii) The networks which are analysed may contain non-linear resistive and non-linear reactive elements. This implies that such elements as surge diverters, saturating reactors and voltage dependant capacitors may be included.

(iii) Parameters may be made conveniently time dependent. This feature provides a very valuable facility since it enables switching operations to be simulated, or permits the generation of pulse trains or other complicated forcing and switching functions.

(iv) The method readily permits initial conditions to be simulated with no loss of accuracy. The majority of typical studies are carried with the assumption that the network is initially de-energised. This implies that all currents and voltages start off at zero and that there is thus no stored energy anywhere. This provision to be able to specify non-zero initial values is a useful one especially for cases where transient perterbations of the steady state performances of networks are being analysed.

(v) By comparison with other techniques the method is economical in computer storage and time. Indeed the latest 'history analysis' technique used here - which will be described later - produces a considerable improvement in operating speed over the basic method which is itself at least as fast as any of its rivals.

(vi) The mechanics of the process of computation are such that currents as well as voltages are available at all nodes in the network at each interval of time. This fact represents an advantage over techniques such as the lattice method where currents, if required, have to be determined by a separate computation.

Naturally the method has certain drawbacks and the more prominent among these are as follows:-

(i) The frequency dependence of the system parameters - especially of transmission line parameters - cannot be taken into consideration.

(ii) The method demands for its operation that the network be divided such that the time for a disturbance to travel between two adjacent nodes is the same anywhere in the system. This usually means that additional artificial nodes have to be introduced into long transmission lines to enable this requirement to be met. The introduction of these new nodes itself produces problems. In the first place the travel (or 'transit') time between any two nodes in the modified system has to be the highest common factor of all the transit times of the lines in the original network - before the addition of the new nodes. This factor may, in practical cases, be a very small value implying that a large number of additional nodes have to be incorporated - so many, in fact, that the storage capacity of the computer used is exceeded. Practical programs - such as SUSAN - thus have to incorporate a 'compromise' facility to keep the number of nodes added within reasonable limits. This procedure naturally provides a source of inaccuracy.

As the number of additional nodes which have to be added to the network increases, there is a corresponding decrease in the transit time between any two which are adjacent. The transient calculation is performed using what is effectively an iterative process - in the sense that the whole network is scanned, node by node, at a given time from the commencement of the transient, and this scanning is repeated at intervals corresponding to the basic transit time (the transit time between two nodes). It therefore follows that an increase in the number of additional nodes not only necessitates a longer scan time - with more nodes to be handled - but also means more scans for a given study length. In general, therefore, the computer time taken for a typical problem varies - very approximately - as the square of the total number of nodes necessary to describe the network. This 'square law' has, however, been modified in the SUSAN system to a more favourable figure. The modification is effected by the use of 'past history' analysis. This is described in detail on page 68.

(iii) Lumped reactive elements may be handled using the graphical method, but only at the expense of some approximation. It has already been mentioned that the model used for, say, a lumped inductor is that of a short circuited transmission line having a finite transit line. Such models are, of course, used widely in other transient evaluation schemes and thus the disadvantages which they afford are not peculiar to the graphical method.

3.3. The Basic Technique

The graphical method may be explained in a variety of ways. Fundamentally it amounts to an extension of the 'method of characteristics' long used for the solution of differential equations of the kind considered in this field. The clarity of the process is considerably enchanced if Bergeron's concept of 'observers' moving along the transmission lines is retained. It is this clarity which renders the technique so valuable as a teaching tool.

The origins of the analysis follow similar lines to those of the lattice method which has already been briefly considered. Here again lines are considered to be lossless, though losses may be simulated by the inclusion of series and shunt lumped resistive elements. The D'Alembert solutions of the wave equations are again adopted in the same form as they were described in equations 2.5 and 2.6 We thus have:-

$$i = F1.(x - at) + F2.(x + at) \quad \ldots\ldots\ldots 3.1.$$

$$v = Z.F2.(x - at) - Z.F2.(x + at) \quad \ldots\ldots\ldots 3.2.$$

where i and v are the current and voltage at any point distance x from the zero reference at time t.

As mentioned before the equations for voltage and current may be interpreted as describing travelling waves. In the case of the lattice technique this interpretation is retained and the process consists of computing the nett algebraic effect of these waves at particular points of interest.

In the graphical method this idea of travelling wave motion is deliberately discarded. It is not sufficient to describe the graphical technique simply as an alternative formulation of the D'Alembert equations to that employed by Bewley. Nor, on the other hand, is there any real advantage to be gained by attempting to combine the two techniques as was done by O'Kelly[36]. There exists an essential difference in the basic philosophy of the two methods which may be readily overlooked.

The techniques may be developed as follows. Equation 3.1. is multiplied throughout by Z and successively added to and subtracted from 3.2. This operation yields:-

$$v + Z.i = 2.Z.F1.(x - at) \quad \ldots\ldots\ldots\ldots 3.3.$$

and

$$v - Z.i = 2.Z.F2.(x + at) \quad \ldots\ldots\ldots\ldots 3.4.$$

If, in equation 3.3 the right hand side may be considered constant, then the left hand side of the equation will also be constant. A similar argument applies to the left hand side of equation 3.4 when (x + at) is constant.

If some observer is considered who moves along the line in the positive x direction with a constant velocity 'a', then we see that for this observer the expression v + Z.i must appear constant. Similarly v - Z.i appears to be constant for an observer moving along the line in the opposite direction with a velocity -a.

For these moving observers, therefore, equations 3.3 and 3.4 may be modified to:-

$$v + Z.i = K1 \quad ...... \quad 3.5.$$

and

$$v - Z.i = K2 \quad ...... \quad 3.6.$$

where K1 and K2 are constant factors. Equations 3.5 and 3.6 may be themselves interpreted as straight lines on the voltage-current plane having slopes of -Z and +Z.

It should be noted that nowhere has it been stated that the moving observers necessarily travel with any voltage or current waves in the network. Indeed the concept of travelling waves as such has already been lost.

If two observers are considered moving as described previously, that is with a velocity 'a', along a transmission line, but in opposite directions, then when they meet at the same point on the line, logic dictates that they must both measure the same voltage and current. The passages of these observers may be considered as being represented by straight lines on the voltage-current plane.

The voltages and currents at certain points within any network will always be absolutely determined or related, irrespective of the general transient situation in the network as a whole. These are nodes at which there are lumped circuit elements such as resistors, voltage or current sources or open or short circuits. The conditions at such nodes may thus be represented on the voltage-current plane by what may be termed 'absolute characteristics'.

An alternative description of the graphical method has been advanced by Arlett[37] and others which is essentially a re-statement of the 'method of characteristics'. Here the concept of wave motion is retained though the actual graphical techniques which are employed are identical to those described here. It is felt that the essential simplicity of this method is somewhat lost by a rather formal treatment of this kind, though one useful concept, that of points on the voltage-current plane representing 'states of power' progressing through the network, does emerge. A wave progressing through the network can be represented by a state of power (v x i), though the same concept can be employed to describe network conditions in the steady state - for example the state of power in a de-energised network with no stored energy is everywhere zero.

3.4. An Example

A technique of this kind is best illustrated by reference to a typical example. It is not considered desirable, in a work of this kind, to labour individual aspects of the method - for example the way in which complex forcing functions are handled. Previously referred to publications by the writer and his colleagues cover the majority of the salient points and do not need repetition. In any event aspects of the technique will be amplified in subsequent chapters when the modification of the process to a form suitable for computer application is examined.

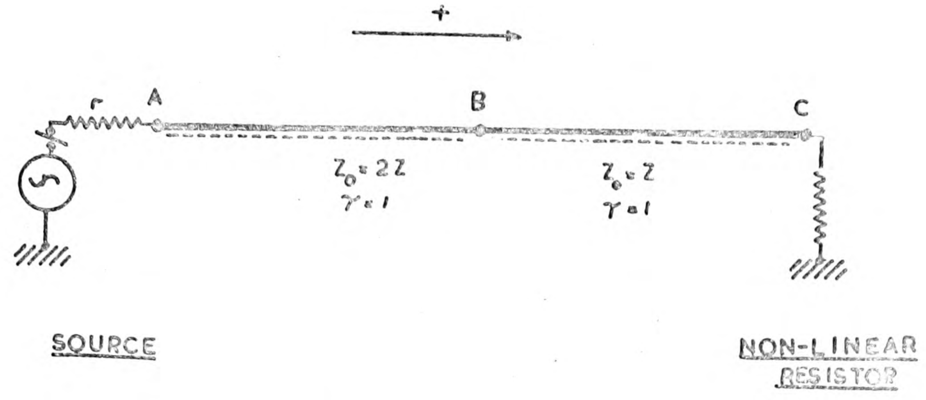
It is therefore convenient to consider an example which incorporates a number of 'complex' features all at the same time. Such a situation is illustrated in fig. 3.1. This example, albeit somewhat artificial in character, nevertheless illustrates clearly the power of the graphical method. Two transmission lines, AB and BC are connected in series with their junction at B. It is assumed that the transit times of the two lines are equal, but that their surge impedances are respectively 2Z and Z. The lines are assumed to be lossless.

At time zero the previously de-energised system is excited by a sinusoidal forcing function applied at A - this is equivalent to closing the switch at A. The forcing function voltage source has an internal impedance of 'r' which is assumed to be resistive.

The line BC is loaded at C with a non-linear resistor - in practice such non-linearities are frequently found in many types of network ranging from surge diverters in power systems to devices with non-linear input impedances in electronic circuits.

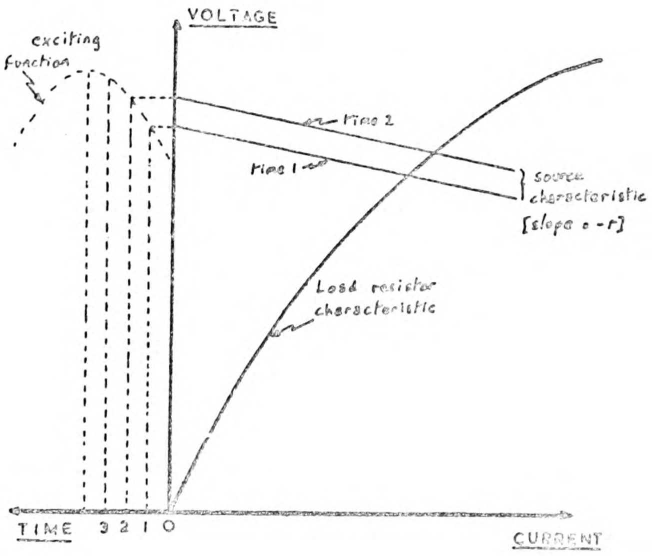
The graphical method permits the simultaneous computation of transient voltages and currents at definite increments of time after time zero. Construction is carried out using a combined voltage-current and voltage-time plane as shown in fig. 3.2. In this figure the 'absolute characteristics' of the non-linear resistance load, and the family of absolute characteristics representing the exciting function at various instants in time are shown. In the case of simpler forcing functions - step functions, for example, the necessity for a voltage-time plane is removed since the source voltage becomes time invariant.

The construction itself is illustrated, in part, in fig. 3.3. This shows the voltage-current and voltage-time planes as in fig. 3.2, but with the addition of lines representing the passages of observers in the network.



+
r
A
B
C
$Z_0 = 2Z$
$\gamma = 1$
$Z_0 = Z$
$\gamma = 1$
SOURCE
NON-LINEAR
RESISTOR

FIG. 3.1.



VOLTAGE
exciting
function
time 2
time 1
source
characteristic
[slope = -r]
Load resistor
characteristic
TIME
3 2 1 0
CURRENT

FIG. 3.2.

It is necessary to imagine that each of the two transmission lines has an observer free to travel along it and to continually monitor voltage and current conditions. Such an observer travels with the same velocity as a travelling wave would achieve on the line, though this does not imply that the observer actually travels with any wave. For convenience we may assume that the unit of time basic to this problem equals the transit time (one way) of either of the transmission lines. Assume also that at the instant of switching the sinusoidal source voltage is some little way below its positive peak value.

At time -1, one time unit before switching, the voltages and currents throughout the system are everywhere zero. Thus an observer on line AB, stationed at B, would record zero voltage and current and these conditions may be plotted on the voltage-current plane.

Now imagine that the observer on AB moves towards A - he is therefore moving in the negative reference direction as defined in fig. 3.1. - and arrives at time zero, at the instant that the forcing function is applied. The passage of the observer, who, moving at wave velocity sees that the voltage and current which he monitors are always directly related by the surge impedance of the line (eqn. 3.6) may be represented on fig. 3.3 by the line $B_{-1}A_0$. The notation here is that of a letter denoting the node together with a subscript giving time information. Thus A_2 refers to the conditions at node A at time 2.

The point A_0 is determined by the intersection of the line $B_{-1}A_0$ with the absolute characteristic line corresponding to the output of the source at time zero (line PA_0 in fig. 3.3.). The initial voltage and current at A at time zero may thus be read directly from the graph.

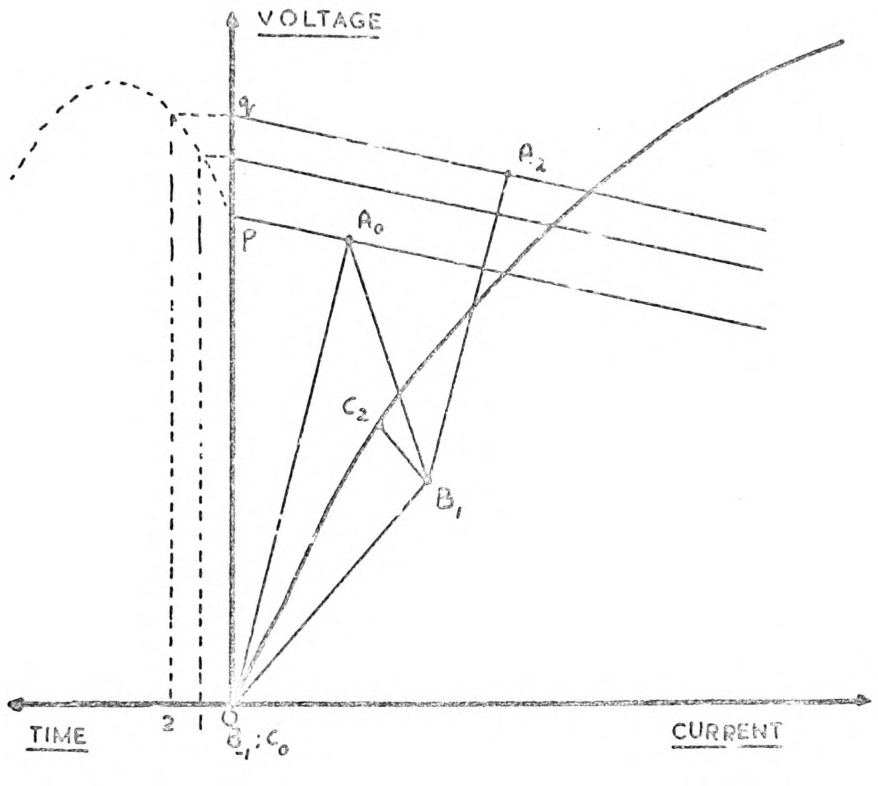


FIG.3.3.

One of the characteristics of the graphical method is apparent here, this is the necessity to have information on the past history of the network before the transient disturbance is applied. In practice such information is normally available and therefore this requirement does not present any real problems.

At C at time zero, the voltage and current are still zero since the disturbance initiated at A at this time has not yet had an opportunity to propagate through the network. An observer stationed at C would thus record both zero voltage and current. Now suppose that this observer moves towards B at the wave velocity, simulatneously with the observer on line AB. Since the two observers started their journeys from C and A respectively at time zero, they will thus meet exactly at B at time 1. When they meet they must measure the same voltage and current values.

The passage of the observer from C may be represented by the line C_0B_1 in fig. 3.3, whilst that of the observer on AB is represented by A_0B_1. The intersection of these two lines thus gives the required voltage and current conditions at B at time 1. The slopes of the two characteristic lines differ since the two observers are travelling in different directions relative to the positive reference direction defined for the problem.

The construction then continues as follows. The observer on BC immediately reverses his direction of travel, leaving B at time 1 and arriving back at C at time 2. Here he meets the non-linear terminating resistor whose absolute characteristic has already been drawn on the voltage-current plane. The intersection of the characteristic line representing the passage of the observer (B_1C_2) and the absolute characteristic line thus yields the point C_2 which gives the voltage and current conditions at C at time 2.

The observer on AB, for his part, retraces his steps from B, arriving at A at time 2. The point A_2 is thus produced by the intersection of the characteristic line for the observer with the absolute characteristic of the source (qA_2). This line differs from the source characteristic line at time zero (pA_0) simply because the source voltage has altered in the intervening period. The construction is then continued in a similar manner for a long as required.

Graphs of voltage and current against time may be prepared from the points located on the voltage-current plane. It will be noted that information is available about the conditions at each node in the network at intervals of twice the basic system time unit - corresponding to twice the transit time of any line in the network. ~~at intervals of twice the basic system time unit - corresponding to twice the transit time of any line in the network.~~ Drawing a curve between these points can be largely a matter of experience since in some cases the transient will take the form of a smooth curve, whilst in others it may have pronounced discontinuities. Additional points on the graphs may be obtained by repeating the construction either with additional nodes inserted in the network - thus making the construction much more complex but at the same time reducing the basic network transit time, or by repeating the construction exactly as detailed above, but choosing a different starting time.

Fig. 3.4. shows an example of the construction of fig. 3.3 repeated, but instead of starting from C at time 0, and from B at time -1, out observers start from C at time 0.5 and from B at time -0.5. The observer from B at time -0.5 will thus reach A at time 0.5, that is after the transient has commenced. Even though this observer did not, in this case, arrive at the source just as the disturbance was being initiated, as in the case of fig. 3.3, nevertheless there is no loss of accuracy resulting from this new procedure.

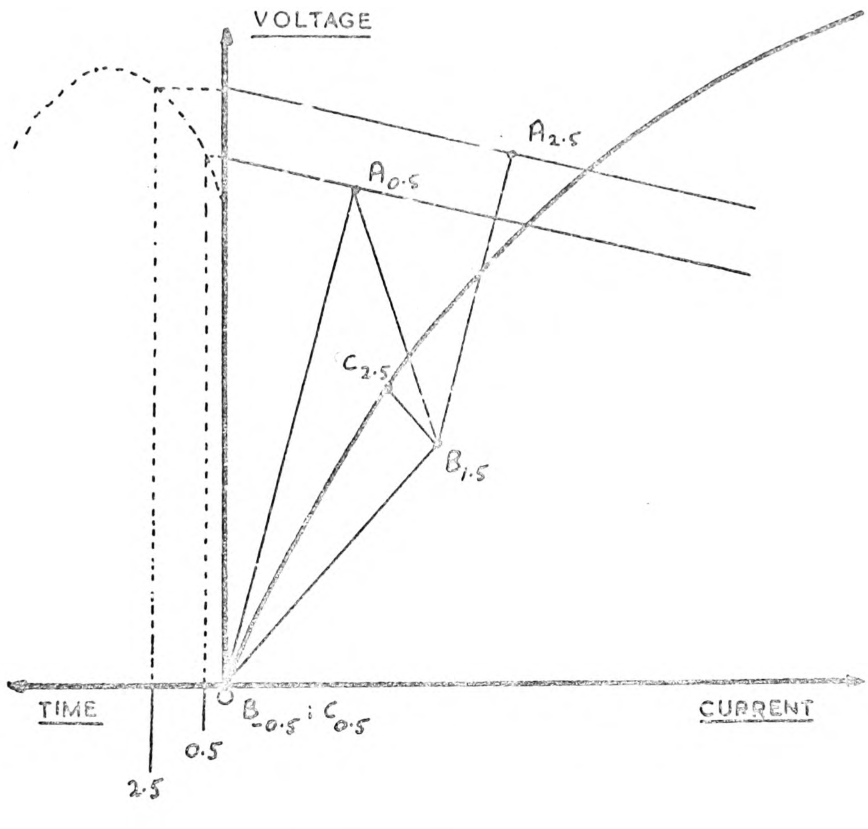

VOLTAGE
$A_{2\cdot 5}$
$A_{0\cdot 5}$
$C_{2\cdot 5}$
$B_{1\cdot 5}$
TIME
$B_{-0\cdot 5}$; $C_{0\cdot 5}$
CURRENT
0.5
2.5

FIG. 3.4.

By combining results from figs. 3.3 and 3.4., voltage and current conditions at each of the nodes of interest within the network are available. Results are present at A at times 0, 0.5, 2, 2.5, 4 etc., while results at, say C have been obtained for times 2, 2.5, 4, 4.5 and so on. Obviously the effect of the transient disturbance cannot be felt at C before time 2 due to the propagation delays in the network.

These basic principles of analysis may be extended to deal with a variety of other situations including such network configurations as the junction of a number of transmission lines, the inclusion of series and shunt lossy elements within lines and the loading of lines with lumped reactive elements. It has already been mentioned that in this last case a degree of approximation is involved and this will be discussed in a subsequent chapter.

As one final illustration of this basic graphical application of the method, we may consider the construction which is adopted for the case of a series resistor between two lengths of transmission line. This particular circuit is valuable for the simulation of line losses, remembering that the method pre-supposes that all transmission lines are lossless. The circuit under analysis is shown in fig. 3.5. We may assume that this forms a section of a larger network and that the network is energised. The two sections of line each have the same characteristic impedance, and each has the same transit time which equals the basic unit of time for the problem.

Suppose that the conditions at A and D are known at some time 't'. The problem is thus to determine the voltages and currents present at B and C one time unit later, that is at time (t + 1). Since it is assumed that the lumped series resistor, R, has zero transit time, it therefore follows that the

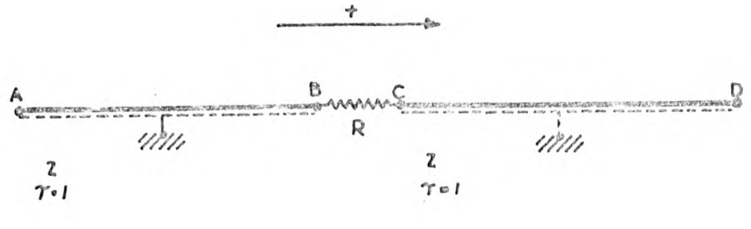

+
A
B
C
D
R
Z
τ=1
Z
τ=1

FIG.3.5.

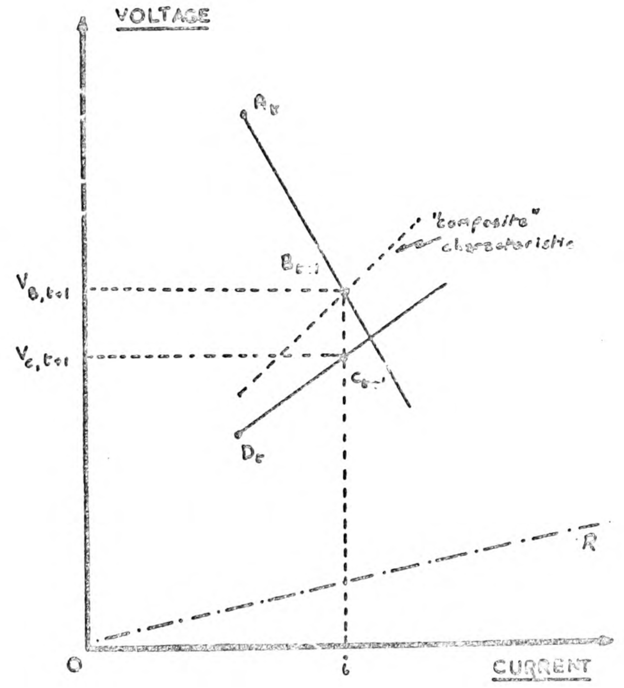

VOLTAGE
"composite" characteristic
CURRENT
O
R

FIG.3.6.

current entering and leaving it is the same it all times. The voltage at B will differ from that at C, only equalling it when the current through the resistor is zero. The construction used to handle this case is shown in fig. 3.6.

The observer travelling from A in the positive reference direction is represented by the line, whose slope is $-Z$, A_tB_{t+1}. The point B_{t+1} is located as the intersection of this line with the 'composite characteristic' which is itself formed as the series sum of the resistor characteristic (R) and of the characteristic representing the passage of the observer from D in the negative reference direction. This last characteristic also has a slope of Z. A vertical projection of the point B_{t+1} yields the point C_{t+1} - this operation fulfills the condition that the currents at each side of the resistor shall always be equal. The current through R is thus shown on the figure as 'i', whilst the voltages at either end are given by V_{Bt+1} and V_{Ct+1} respectively.

It may be appreciated that even though the basic techniques involved in this type of graphical construction are relatively straightforward in themselves, the analysis for systems having any degree of complexity rapidly becomes extremely complicated. Accuracy is, of course, also limited by the very nature of the drawing technique, and by the fact that errors tend to be cumulative. We may now consider, therefore, the adaptation of the graphical method to a form more suitable for computer processing.

3.5 Accuracy and Cost Effectiveness of the Graphical Method

In general the accuracy of a process such as the graphical method is largely dependent on the constraints placed upon it by the network under investigation. Thus for networks consisting only of lossless transmission lines, having no energy storage elements (capacitors or inductors), and subject to step excitations, the graphical method can be said to produce results which are entirely accurate. In the case of networks having lines with significant attenuation, errors tend to result even though these can be minimised by, for example, the inclusion of line series resistors to simulate losses.

Whilst it is therefore difficult to produce absolute data on the accuracy of the process when applied to networks in general, it is, however, quite possible to make comparisons between the graphical method and alternative techniques when they are used for the solution of the same problem. In the absence of readily available production programs based on these alternative methods direct comparisons are difficult. However the work of Bickford and Doepel[10] is of importance here since they were able to make comparative assessments of the performance of programs based on the lumped parameter technique, the Bewley lattice method and the Fourier transform approach when applied to the solution of a class of restriking voltage transient problems.

Though their comparisons did not involve the graphical method directly, it is suggested that the Bewley lattice method and the graphical method have the same basis and employ the same approximations. Thus similar results would have been obtained if the graphical rather than the lattice method had been employed. Computing times tend to be similar for the two approaches over a wide range of applications.

The lumped element method is of value only in cases where the lengths of transmission lines are relatively small. Comparisons were thus made between this technique and the lattice method for a restriking transient following a 'short line' fault. Source inductances and line losses could be neglected and thus the lattice method could be regarded as giving accurate results. The lumped element method, on the other hand, produced errors amounting to 2.7% of the peak transient voltage across the circuit breaker, together with an error of some 4.2% in the time at which this peak occurred.

The efficiency - or in practical terms the cost-effectiveness - of the methods can be assessed by estimating the likely cost of runs using the various techniques. Simply comparing the run times is not valid, except in the case of small computers, since the amount of fast storage required for a program is of importance in computing the cost when large computers equipped with multi-programming facilities are employed. Bewley and Doepel found that for the example quoted above the lumped parameter method was almost exactly twice as expensive as the lattice diagram technique.

A similar problem, only this time involving a much longer length of transmission line (some ten miles) was used to compare the performance of the lattice method with the Fourier program. In this case the line was assumed to be subject to earth penetration effects (i.e. had an imperfect earth) which could be handled by the Fourier program. The difference in the results showed only a 5% discrepancy between the peak voltages computed by each program, though the cost of the more accurate Fourier solution was some four times that of the lattice diagram result.

3.6 Alternative Explantion of the Graphical Method

The explantion offered previously for the graphical method makes use of the concept of hypothetical observers moving along transmission lines. Whilst this approach results in a useful physical interpretation of the method, the necessity for these imaginary observers is open to criticism. The method can, however, be explained without recourse to the observer principle as follows:-

In equation 3.1, the function F1.(x-at) may be considered as representing a current wave travelling in the forward reference direction. Hence let $F1.(x-at) = i_f$. Similarly a 'backward' wave, i_b, is represented by F2.(x+at).

Hence equation 3.1 becomes:-

$$i = i_f + i_b \qquad \ldots\ldots\ldots\ldots 3.1(a)$$

simularly $$v = Z.i_f + (-Z).i_b = v_f + v_b \qquad \ldots\ldots\ldots\ldots 3.2(a)$$

From a comparison of equations 3.1(a) and 3.2(a), bearing in mind that the current wave i_f is associated with the voltage wave v_f, we see that:-

$$v_f/i_f = Z \qquad \ldots\ldots\ldots\ldots 3.7$$

and $$v_b/i_b = -Z \qquad \ldots\ldots\ldots\ldots 3.8$$

If we now consider small changes in, say, v_f accompanied by small changes in i_f, we see that:-

$$v_f = Z.\ i_f \qquad \ldots\ldots\ldots\ldots 3.9$$

Now a point in the v-i plane represents the voltage and current at a particular point in a network at a specific time. It represents what has been previously called a 'state of power'.

Equation 3.9 may thus be interpreted as implying that a change in the state of power at a given point in a line can only take place along a straight line of slope $\pm Z$ in the v-i plane.

As a result of the arrival of a disturbance at a point on the line, the state of power at that point changes from its original value (v1,i1) to a new value (v2,i2). This change is made along a straight line on the voltage current plane whose slope is $\pm Z$. The magnitude of that change is dependent on the new boundary conditions at that point produced by the arrival of the disturbance. The new voltage and current (v2, i2) is thus determined by finding the intersection of a line of slope Z passing through the previous state of power, with a second line representing the boundary conditions at the time fo arrival of the disturbance. This operation forms the basis of the graphical method whose practical operation has already been discussed in some detail.

4. FORMULATION FOR COMPUTER ANALYSIS

4.1 The concept of 'reference direction'

In the previous chapter the basic graphical method was examined. Here it was necessary to specify in each case a positive and negative reference direction along each of the lines forming a part of the network. In the cases which were considered this was something of a formality since these were all 'linear' in the sense that they contained no meshes or junctions of more than two branches.

Networks which have junctions of three or more branches present problems from the topological point of view if this idea of reference direction is retained. For example the network shown in fig. 4.1a is different from that shown in fig. 4.1b. Early programs written by the writer and others did use this reference direction idea with the result that data preparation became a relatively difficult matter, prone to error, and that the program was rather restricted in its application.

It is therefore advantageous to abandon the definition of a specific positive reference direction for each line. Nevertheless the direction of a current is as important as its magnitude and it is thus necessary to establish a means of identifying current direction. It is convenient to assume that currents entering nodes are always considered positive. Such a policy naturally involves certain modifications to the basic method discussed so far as follows :-

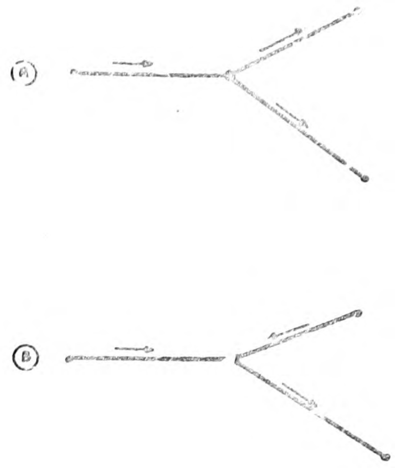


FIG.4.1.

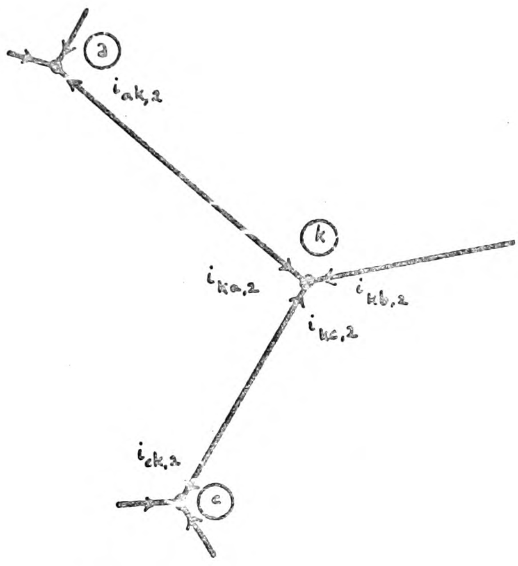


FIG.4.2.

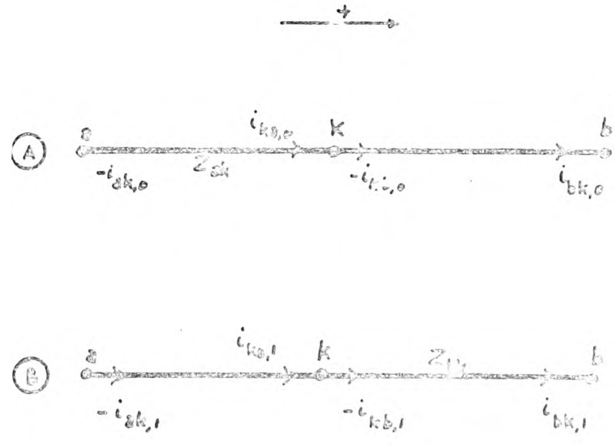


FIG.4.3.

4.1.1 Notation

A convenient notation to describe voltages and currents is:-

(a) For voltages:- $V_{n,t}$ where 'n' is a node number, 't' is a time

Voltages are measured with respect to the ground plane in all cases.

(b) For currents:- $i_{ak,t}$ where 'a' is node into which current i flows, 'k' is node from which current has passed and 't' is time as before.

A diagram illustrating the use of this notation, showing the currents in a network at time 2, is given in fig. 4.2.

(c) Transmission lines:-

Lines are denoted by the nodes which they connect. Thus the line between nodes 'a' and 'k' is line ak. Though 'internal' numbering of lines is carried out automatically by the computer during execution of the program, specific numbering by the user is not required.

Line surge impedances are identified by two subscripts corresponding to the line which they represent. Thus Z_{ak} is the surge impedance of line ak. Line resistance is treated in a similar manner.

4.2 The Solution Equations

Fig. 4.3 (a) shows a representation of two transmission line sections ak and bk having a junction at k. The transit times of each of the sections are equal and their surge impedances are Z_{ak} and Z_{bk} as shown. The figure shows also the currents flowing at time zero and this diagram serves to reconcile the concept of positive reference direction for the network as a whole (shown by the long arrow) with the new policy of considering currents as positive when they enter nodes. Fig. 4.3 (b) describes the same network, though in this case the currents are shown one time unit later at time 1.

The graphical construction necessary to determine the conditions at node k at time 1 knowing them at nodes a and b one time unit previously is shown in fig. 4.4. Currents corresponding to those shown in fig. 4.3 are noted on this diagram.

The point k_1 is determined by the intersection of the two characteristic lines shown or, more formally, by the solution of the simulatneous equations which represent each of time. These equations are, from fig. 4.4:-

<u>line $a_o k_1$</u>

$$(V_{a,0} - V_{k,1}) = -Z_{ak} \cdot (-i_{ak,0} - i_{ka,1})$$

thus:- $$V_{k,1} + Z_{ak} \cdot i_{ka,1} = V_{\cancel{k}\,a,0} - Z_{ak} \cdot i_{ak,0} \quad \ldots\ldots 4.1$$

<u>line $b_o k_1$</u>

$$(V_{k,1} - V_{b,0}) = Z_{bk} \cdot (-i_{kb,1} - i_{bk,0})$$

thus:- $$V_{k,1} + Z_{bk} \cdot i_{kb,1} = V_{b,0} - Z_{bk} \cdot i_{bk,0} \quad \ldots\ldots 4.2$$

In the general case the junction within the network will consist of a joining of 'n' lines, and here the corresponding equations for all lines become:-

$$\begin{bmatrix} Z_{ak} & \cdots & & 1 \\ \vdots & Z_{bk} & \cdots & 1 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots\ Z_{nk} & 1 \end{bmatrix} \cdot \begin{bmatrix} i_{ka,1} \\ i_{kb,1} \\ \vdots \\ i_{kn,1} \\ V_{k,1} \end{bmatrix} = \begin{bmatrix} V_{ao} - Z_{ak} \cdot i_{ak,0} \\ V_{bo} - Z_{bk} \cdot i_{bk,0} \\ \vdots \\ V_{no} - Z_{nk} \cdot i_{nk,0} \end{bmatrix} \quad \ldots\ldots 4.3$$

If the node in question is merely a simply junction between transmission lines, (fig. 4.5), we have the additional information that:-

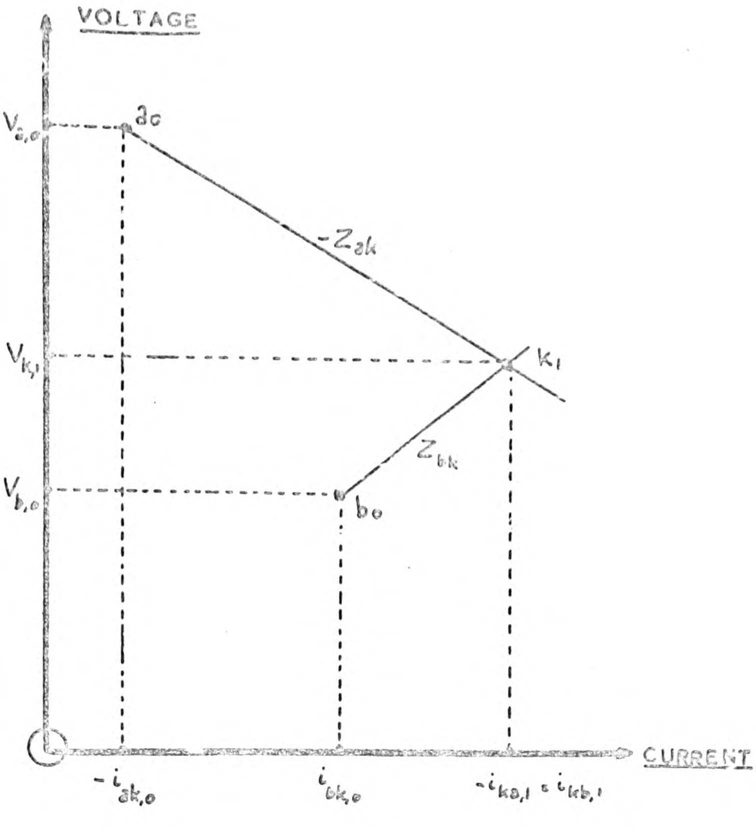

VOLTAGE
$V_{a,o}$
a_o
$-Z_{ak}$
$V_{k,1}$
k_1
Z_{bk}
$V_{b,o}$
b_o
CURRENT
$-i_{ak,o}$
$i_{bk,o}$
$-i_{ka,1} = i_{kb,1}$

FIG.4.4.

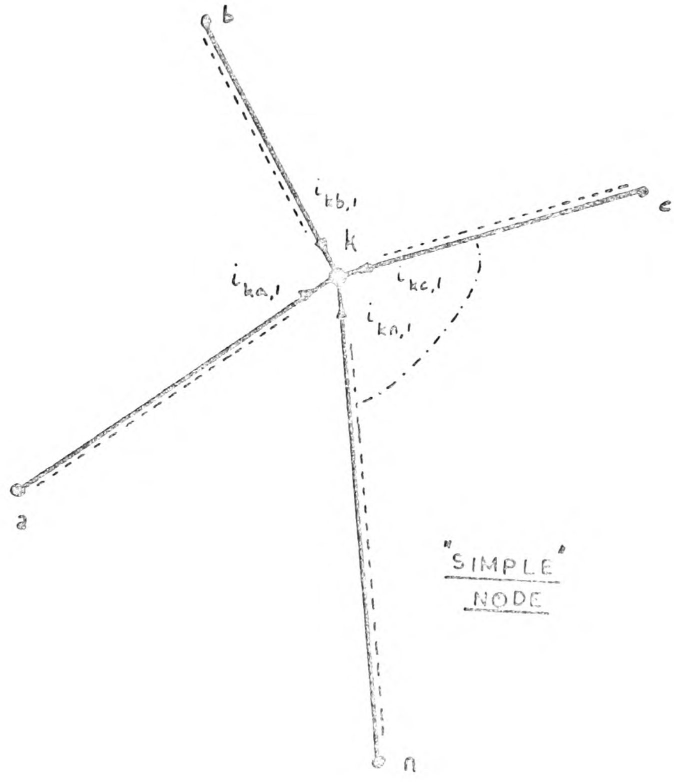

b
$i_{kb,1}$
k
c
$i_{ka,1}$
$i_{kc,1}$
$i_{kn,1}$
a
"SIMPLE" NODE
n

FIG.4.5.

$$i_{ka,1} + i_{kb,1} + i_{kc,1} + \dots + i_{kn,1} = 0 \quad \dots\dots 4.4$$

In order to make the analysis as general as possible, it is convenient to consider the more complex case of a junction between transmission lines where there is some non-reactive shunt element at the node. This idea is shown in fig. 4.6. The shunt element may be resistive (non-linear or linear) or an active source or some combination of both of these. The question of shunt inductance or capacitance placed at the node will be dealt with later.

Equation 4.4 is now modified as follows:-

$$i_{ka,1} + i_{kb,1} + i_{kc,1} + \dots + i_{kn,1} + A.V_{k,1} = B \quad \dots\dots\dots 4.5$$

In this equation 'A' and 'B' are dependent on the nature of the shunt element at the node.

Combining equations 4.3 and 4.5 yields:-

$$\begin{bmatrix} Z_{ak} & 0 & \cdots & 1 \\ 0 & Z_{bk} & \cdots & 1 \\ \vdots & \vdots & \ddots & \\ 0 & & Z_{nk} & 1 \\ 1 & 1 & \cdots 1 & A \end{bmatrix} \cdot \begin{bmatrix} i_{ka,1} \\ i_{kb,1} \\ \vdots \\ i_{kn,1} \\ V_{k,1} \end{bmatrix} = \begin{bmatrix} V_{ao} - Z_{ak} \cdot i_{ak,o} \\ V_{bo} - Z_{bk} \cdot i_{bk,o} \\ \vdots \\ V_{no} - Z_{nk} \cdot i_{nk,o} \\ B \end{bmatrix} \quad \dots\dots 4.6$$

It is convenient to simplify the right hand side of equation 4.6 by making the substitutions:-

$$V_{a,0} - Z_{ak}.i_{ak,0} = a_0$$

$$V_{b,0} - Z_{bk}.i_{bk,0} = b_0 \quad \dots\dots 4.7$$

$$\dots\dots$$

$$\dots\dots$$

$$V_{n,0} - Z_{nk}.i_{nk,0} = n_0$$

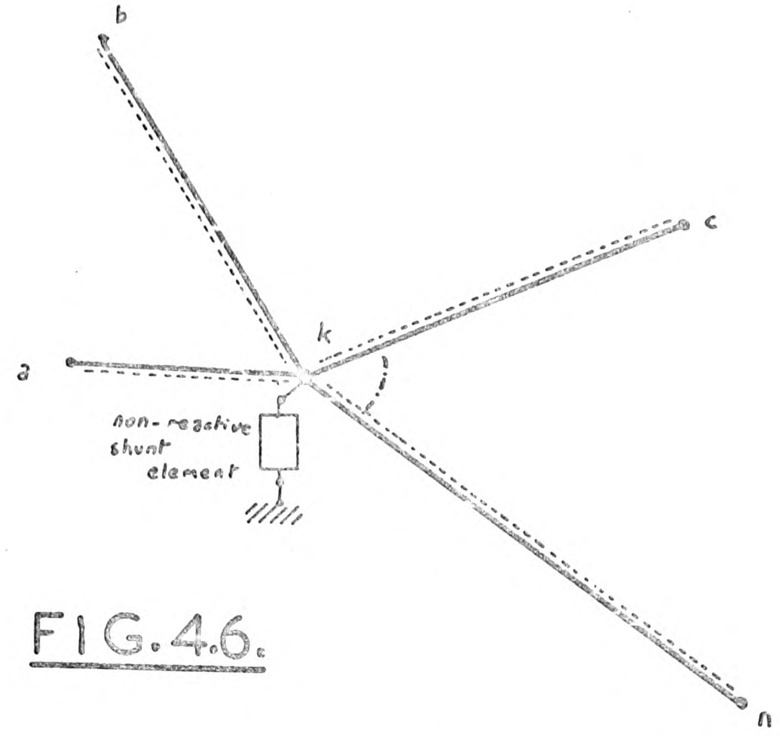


FIG.4.6.

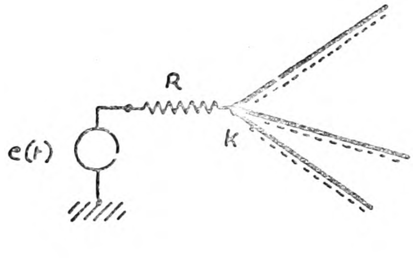


FIG.4.7.

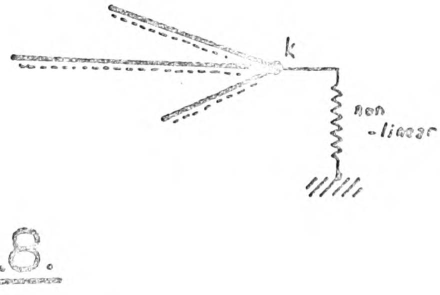


FIG.4.8.

The right hand side thus becomes:-

$$\begin{pmatrix} a_0 \\ b_0 \\ c_0 \\ \cdot \\ \cdot \\ \cdot \\ n_0 \\ B \end{pmatrix} \quad \ldots.\ 4.8$$

4.3 <u>'A' and 'B'</u>

The terms A and B in equations 4.6 have values dependent on the boundary conditions at node k.

4.3.1 <u>Case 1 - Connection node</u>

This is a simple interconnection of lines at k (fig. 4.5). The boundary condition at k is given by equation 4.4 and it thus follows that:-

$$A = B = 0$$

The equations for solution thus become:-

$$\begin{bmatrix} Z_{ak} & 0 & 0 & \cdots & 1 \\ 0 & Z_{bk} & 0 & \cdots & 1 \\ 0 & 0 & Z_{ck} & \cdots & 1 \\ \vdots & \vdots & & \ddots & \\ & & & Z_{nk} & 1 \\ 1 & 1 & 1 \cdots & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{ka,1} \\ i_{kb,1} \\ i_{kc,1} \\ \vdots \\ i_{kn,1} \\ V_{k,1} \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ \vdots \\ n_0 \\ 0 \end{bmatrix} \quad \ldots..\ 4.9$$

These may be solved for $V_{k,1}$ and back-substituted for the currents at time 1 ($i_{ka,1}$ etc.). One method of solution follows:-

Expanding equation 4.9 we have:-

$$Z_{ak} \cdot i_{ka,1} + V_{k,1} = a_0$$

$$Z_{bk} \cdot i_{kb,1} + V_{k,1} + b_0$$

$$\vdots$$

$$Z_{nk}.i_{kn,1} + V_{k,1} = n_0$$

$$i_{ka,1} + i_{kb,1} + \ldots + i_{kn,1} = 0$$

From these:-

$$i_{ka,1} = (a_0 - V_{k,1})/Z_{ak} = a_0/Z_{ak} - V_{k,1}/Z_{ak}$$

Similarly:-

$$i_{kb,1} = b_0/Z_{bk} - V_{k,1}/Z_{bk}$$

Sunstituting these results into equation 4.4 we have the relation:-

$$(a_0/Z_{ak} - V_{k,1}/Z_{ak}) + (b_0/Z_{bk} - V_{k,1}/Z_{bk}) + \ldots + (n_0/Z_{nk} - V_{k,1}/Z_{nk})$$
$$= 0$$

The required voltage, $V_{k,1}$, may be extracted directly from this relation as:-

$$V_{k,1} = \frac{(a_0/Z_{ak} + b_0/Z_{bk} + c_o/Z_{ck} + \ldots + n_0/Z_{nk})}{(1/Z_{ak} + 1/Z_{bk} + 1/Z_{ck} + \ldots + 1/Z_{nk})}$$

or, alternatively:-

$$V_{k,1} = (a_0.Y_{ak} + b_0.Y_{bk} + c_0.Y_{ck} + \ldots + n_o.Y_{nk}) / Y_{kk} \quad \ldots 4.10$$

In equation 4.10:-

$$Y_{ak} = 1/Z_{ak}$$

$$Y_{kk} = \text{'self surge admittance' of node } k$$
$$= \sum_{s=a}^{n} y_{sk}$$

The currents at the junction may then be determined by simple substitution and, in general we have that:-

$$i_{kn,1} = (n_0 - V_{k,1}).Y_{nk} \quad \ldots\ldots\ldots 4.11$$

<u>4.3.2 Cases 2 and 3, general solution when 'A' and 'B' not zero.</u>

Referring to equation 4.6, it may be readily shown, following a similar argument to that traced above for the case of a simple connection node, that the general solution for the voltage at a node where either A and/or B are not zero has the form:-

$$V_{k,1} = (a_0 \cdot y_{ak} + b_0 \cdot y_{bk} + c_0 \cdot y_{ck} + \dots + n_0 \cdot y_{nk} - B)/(y_{kk} - A) \qquad \dots\dots\dots\dots 4.12$$

4.3.2.1 Case 2 - Node the impressed e.m.f.

A node having an impressed emf between itself and the ground plane is shown in fig. 4.7. This source is represented in all cases as a perfect voltage generator in series with a resistive internal impedance (R). The voltage itself may or may not be time dependent. The boundary conditions at node k are given in this instance by :-

$$R.\ \sum_{j=s}^{n} i_{kj} + e(t) = V_{k,1} \qquad \dots\dots\dots 4.13$$

Comparison of equations 4.13 and 4.5 shows that here:-

$$\left.\begin{aligned} A &= -1/R \\ B &= -e(t)/R \end{aligned}\right) \qquad \dots\dots\dots\ 4.14$$

This analysis also includes the case of a simple resistor to ground at the junction, since in this instance $e(t) = 0$ and we see that $A = B = -1/R$.

A perfect voltage source, that is one whose internal impedance is exactly zero ($R = 0$), obviously leads to indeterminate values for A and B. A restriction is thus placed on the program barring it from handling such situations. The same argument applies to a short circuit at the node though provision is made in the program for short-circuited nodes. In this case it is only necessary to specify that the nodal voltage is always zero and then compute the currents at the node in the normal way.

4.3.2.2 Case 3 - Node with non-linear resistive shunt element

This case is illustrated in fig. 4,8. It is assumed that the voltage-current characteristic of the non-linear resistor has the form:-

$$i = a.v^{b} + c \qquad \ldots\ldots\ 4.15$$

In addition a 'flashover level' is specified ('d' in fig. 4.9). This facility is useful for the simulation of non-linear devices which incorporate some breakdown mechanism - a power system surge diverter which has a series spark gap together with a non-linear resistive element is an example. When this facility is invoked the non-linear resistor is assumed to remain out of circuit until the nodal voltage exceeds the sparkover level after which time the resistor is effectively switched in and remains in circuit until the current through the element falls to zero. The 'spark gap' is then deemed to have regained its full insulating strength and the process repeats as necessary.

The boundary conditions at the node are illustrated by the equation:-

$$i_{ka,1} + i_{kb,1} + \ldots + i_{kn,1} = a.v_{k,1}^{\ b} + c \quad \ldots\ 4.16$$

From equation 4.16, the relevant values of A and B may be determined by inspection as :-

$$\left.\begin{aligned} A &= 0 \\ B &= a.V_{k,1}^{\ b} + c \end{aligned}\right) \ldots\ldots\ 4.17$$

Letting:- $P = a_0.y_{ak} + b_0.y_{bk} + \ldots\ldots + n_0.y_{nk}$

the solution for $V_{k,1}$ in this case becomes:-

$$V_{k,1} = (P - c - a.v_{k,1})/y_{kk} \quad \ldots\ldots\ 4.18$$

This form of equation (4.18) is best solved by an iterative process. The criterion for simple iteration to be effective is that if the equation is:-

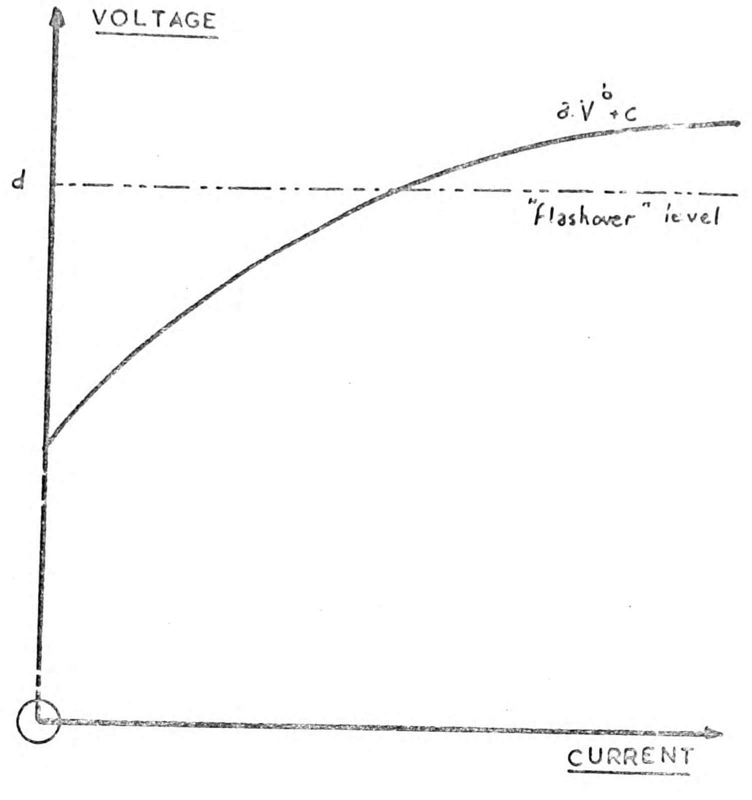


FIG.4.9.

$$f(x) = 0$$

and this is re-written as:-

$$x = \phi(x),$$

then the first differential of $\phi(x)$ ($\phi'(x)$) must be less than 1.

In our present case:-

$$\phi(V_{k,1}) = (P - c - a.v_{k,1}{}^{b})/y_{kk}$$

$$\phi'(V_{k,1}) = -a.b.V_{k,1}{}^{(b-1)} / y_{kk}$$

which will be less than 1 for all positive values of $V_{k,1}$ and for a a and b positive.

The iterative process may be speeded up if required by the use of any of the usual acceleration procedures. Aitken's 'delta squared' technique, for example, was investigated though convergence in practical cases even with simple iteration was found to be rapid and no real advantages accured from its use. Acceleration procedures are of assistance in any event only if $\phi'(V_{k,1})$ is not less than -1.

The nodal currents are computed as in all the previous cases, the current through the non-linear resistor being determined with reference to its original characteristic equation.

4.4. Line Losses

The analysis thus far has considered that all transmission lines are lossless in the sense that they have neither series resistance nor shunt conductance. Indeed the simplicity of the graphical method rests almost entirely on this fact.

For many applications it remains a reasonable approximation to ignore line losses. This is especially true in cases where the lines are short or where the object of the calculation is to determine the extent of overvoltages within the network. In this last case calculated values will usually exceed those achieved in practice, thus incorporating a 'factor of safety' into the computation.

In many instances, however, losses should be included. Usually, especially in the case of power lines, the shunt conductance tends to be negligable in comparison with the distributed series resistance of the line. It is usually sufficient, therefore, to simulate the line resistance only, though a simulation of shunt conductance can be made if required. One problem which confronts workers in the power systems field is that shunt losses are often largely the result of corona effects, all of which are non-linear. It is conceivable that some success might be achieved in the simulation of corona type losses using non-linear shunt resistors though little work has been reported in this area.

The present program has the facility to simulate series line resistance automatically. This is done by the inclusion of lumped resistive elements at intervals along each line such that the sum total of each resistor section equals the total line resistance. The idea is illustrated in fig. 4.10. The number of elements which are used corresponds to the number of sections into which the line is divided as a consequence of the automatic sectioning procedure which will be discussed later.

Since, in the real network, transmission line resistance is a distributed parameter, it would appear logical to assume that the best approximation for series resistance would be to use as many resistor sections as possible. The writer's experience, and that of other workers, notably Dommel[38], has, however, shown that there is very little difference in the results obtained by using only a few elements or by using very many. Indeed work by Dommel indicated that there was no significant variation in the results for a ~~treatment~~ transient overvoltage between the case where 3 resistor sections were used and a second case where 300 such sections were included. Similar observations have been reported by several workers, but, as yet, no real explanation has been offered. The writer feels that in general greatest accuracy will be

FIG.4.10.

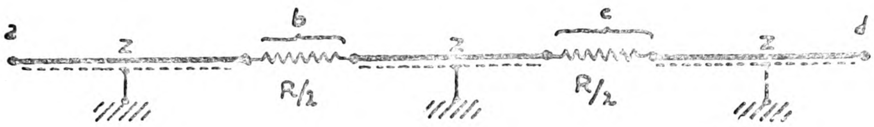


FIG.4.11.

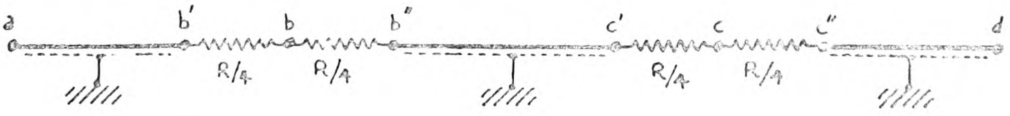


FIG.4.12.

achieved with a large number of sections though the overall effect of line resistance is often quite limited in practical examples.

Incorporation of series resistance elements between two line sections can be accomplished readily as follows:-

4.4.1 Surge impedance modification

Consider a transmission line having a total transit time of 3T. This is subdivided into three equal sections each having a transit time of T. The total line resistance is R and this is simulated by the inclusion of two series elements each having a value of R/2. This situation is shown in fig. 4.11. To assist computation this circuit is now modified slightly to that in fig. 4.12. Nodes b and c are now separated into three parts identified by superscripts as shown.

The solution equations representing line sections ab' and b"c' are:-

$$V_{a,0} - i_{ab',0}.Z = V_{b',1} + i_{b'a,1}.Z \quad \ldots.4.19$$

$$V_{c',0} - i_{c'b'',0}.Z = V_{b'',1} + i_{b''c',1}.Z \quad \ldots.4.20$$

The currents through node b are obviously related by:-

$$i_{b'a,1} + i_{b''c',1} = 0 \quad \ldots..4.21$$

In addition we have that:-

$$\left.\begin{aligned} V_{b'} &= V_b + (R/4).i_{ba} \\ V_{b''} &= V_b + (R/4).i_{bc} \\ V_{c'} &= V_c + (R/4).i_{cb} \\ V_{c''} &= V_c + (R/4).i_{cd} \end{aligned}\right\} \quad \ldots.\ 4.22 \text{ (at any time)}$$

Now since $i_{ab'} = i_{ab}$ etc., equations 4.19 and 4.20 may be modified to read:-

$$V_{a,0} - i_{ab,0}.Z = V_{b,1} + i_{ba,1}.(Z + R/4) \quad \ldots 4.23$$

$$V_{c,0} - i_{cb,o}.(z - R/4) = V_{b,1} + i_{bc,1}.(Z + R/4) \quad \ldots 4.23$$

Nodes b and c thus have their transmission line connection surge impedances modified by R/4 or, in general, by a factor R/(2(n-1) where n is the number of sections into which the line is divided.

The general solution for the voltage at node k remains:-

$$V_{k,1} = (P + (-B))/Y_{kk} + (-A))$$

as before. However if nodes a through n are 'resistive' - that is are similar in type to nodes b and c in the preceding analysis, having series resistive elements incorporated into them, 'p' in the last equation is modified to:-

$$P = \frac{((V_{a,0} - i_{ak,0}.(Z_{ak} - R/(2(N-1))))}{(Z_{ak} + R/(2/N-1)))} + \frac{(V_{b,0} - i_{bk,0}.(Z_{bk} - R/(2/N-1))))}{(Z_{bk} + R/2(N-1)))} + \frac{\ldots}{}$$

('N' is no. of line sections) 4.24

Terms in 'P' which correspond to the equations for line sections between resistive nodes and 'normal' nodes (where a 'normal' node would possibly be a node corresponding to the natural end of a transmission line), are slightly different. Referring again to fig. 4.12 we see that line section ab' is of this type, connecting as it does the normal node a to the resistive node b. In this case the corresponding term in P would be:-

$$\frac{(V_{a,0} - i_{ab,0}(Z_{ab})}{(Z_{ab} + R/2(N-1))}$$

The modification to surge impedance values to allow for line resistance is extended to the computation of the self surge admittance of each node which is 'resistive' in character. Thus we have for normal nodes:-

$$Y_{kk} \doteq \sum_{j=a}^{n} 1/Z_{jk}$$

and for 'resistive' nodes:-

$$Y_{kk} = \sum_{j=a}^{n} 1/(Z_{jk} + mod_{jk}) \quad \text{where } mod_{jk} = R/(2(N-1))$$

In practice, of course, the self surge admittance of a resistive node will have two terms only in the series, since such a node will normally be connected only to two other nodes, forming as it does an intermediate node along a transmission line.

Modification of surge impedances in the manner which has been discussed above is carried out completely automatically by the computer program as and when required in any network under analysis.

4.5 Shunt reactive elements

Any network analysis program such as SUSAN which claims to be general in character must have the ability to handle reactive elements - capacitors and inductors. Consider, in the first instance, such elements connected in shunt at nodes as illustrated in fig. 4.13 The analysis which follows - together with some of that which has appeared earlier - has been published by the writer[39] in abbreviated form, but is repeated here in more detail.

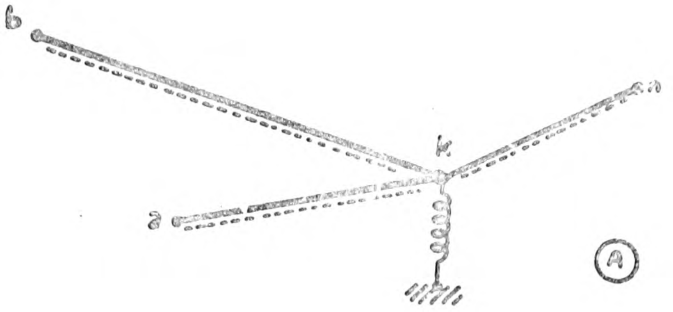
Consider a single inductor conne ted at node k. Currents are as shown in fig. 4.14 If the time varying currents through the inductor is 'i', then:-

$$V_k = L.\frac{di}{dt} \quad \ldots\ldots 4.25$$

where:- $$i = -\sum_{j=a}^{n} i_{kj}$$

Therefore at any time:-

$$V_k = L.d\left(-\sum_{j=a}^{n} i_{kj}\right)/dt \quad \ldots\ldots 4.26$$



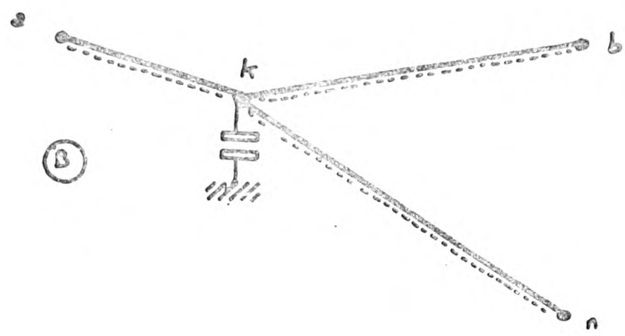


FIG. 4.13.

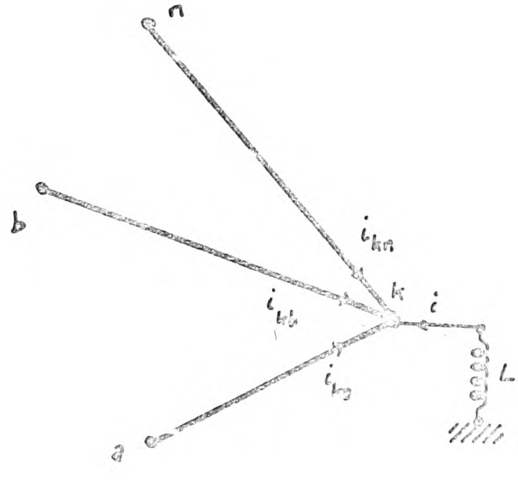


FIG. 4.14.

Using finite differences let $j' = dt$. Additionally let j' equal the basic system transit time (1 time unit). Assuming that the voltage across the inductor varies linearly in the interval from time 0 to time 1, then:-

Average voltage across inductor in the time interval considered

$$= \frac{(V_{k,1} + V_{k,0})}{2} = L.d\left(-\sum_{j=a}^{n} i_{kj}\right)/j'$$

$$= L.\left(\sum_{j=a}^{n} i_{kj,0} - \sum_{j=a}^{n} i_{kj,1}\right)/j'$$

Therefore:-

$$V_{k,1} = \frac{L}{j'/2} \cdot \left(\sum_{j=a}^{n} i_{kj,0} - \sum_{j=a}^{n} i_{kj,1}\right) - V_{k,0} \quad \ldots .4.27$$

The term $L/(j'/2)$ has the dimensions of impedance so let $Z1 = L/(j'/2)$. Rearranging the last equation we have:-

$$(V_{k,1} + V_{k,0}) = Z1 \cdot \left(\sum_{j=a}^{n} i_{kj,0} - \sum_{j=a}^{n} i_{kj,1}\right) \quad \ldots . \; 4.28$$

This may be written:-

$$(V_{k,1} + V_{k,0}) = Z1.(i_1 - i_o)$$

This is, of course the equation of a straight line on the voltage-current plane and this line is illustrated in fig. 4.15.

Triangles AEC and DCE are similar, and thus the slope of line AC is $-$ Z1. Hence the line CB could have been constructed directly from a knowledge of theposition of $V_{k,0}$. Such a construction is shown in fig. 4.16.

Now consider the same system of lines terminated by a short-circuited transmission line of surge impedance Z1 and of transit

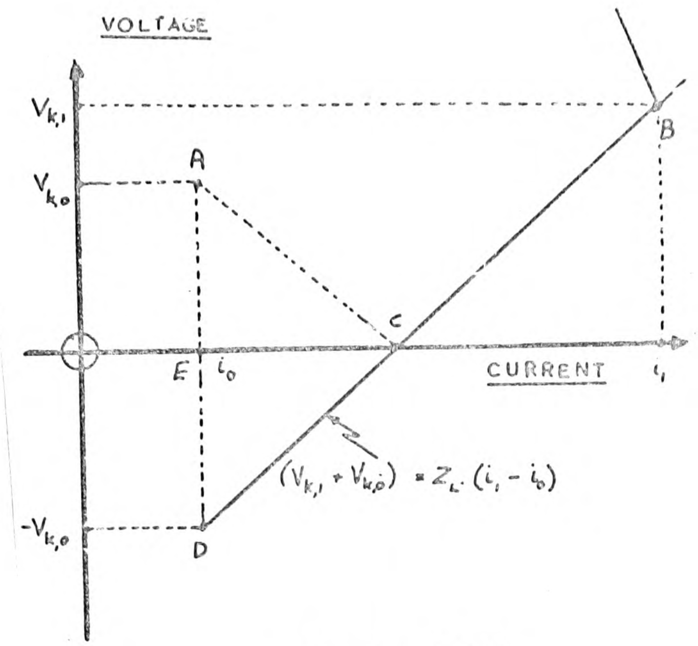


FIG.4.15.

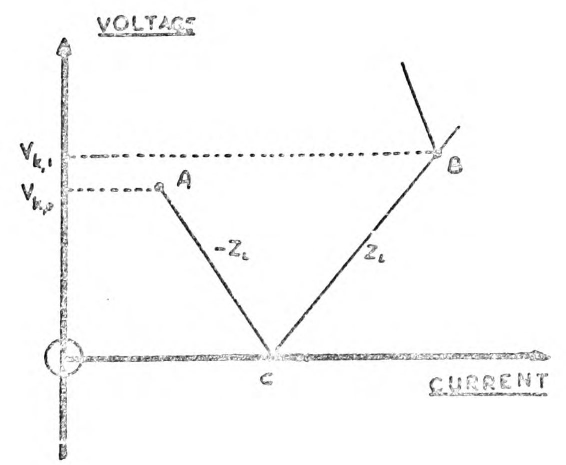


FIG.4.16.

time (j'/2) (fig. 4.17). In this case the graphical interpretation of the passage of observers on this system would be that shown in fig. 4.18. Here characteristic (i) is that seen by an observer moving from k to p during time 0 to j'/2 (0 to ½), whilst (ii) represents the passage of an observer from p to k during the interval ½ to 1.

The equation of characteristic (ii) is:-

$$V_{k,1} + V_{k,0} = Z_1.(i_{kp,1} - i_{kp,0})$$

$$= Z_1.\left(\sum_{j=a}^{n} i_{kj,0} - \sum_{j=a}^{n} i_{kj,1} \right) \quad \ldots. \ 4.29$$

Equations 4.28 and 4.29 are identical, thus showing that the inductor may be represented by a length of short-circuited transmission line whose parameters are related to its inductance.

4.5.1 Shunt capacitors

A similar analysis may be performed for the case of a capacitor connected in shunt at a node (fig. 4.13b). Since it follows almost identical lines to the case of the inductor, it will not be repeated here. It will suffice to say that the element is represented by an open circuited line of surge impedance $Zc = (j'/2)/C$, and of transit time j'/2.

Series connected inductors may also be represented by a length of transmission line in a similar manner to the shunt connected component. Here, however, the line is not short-circuited, but is simply connected into the network so that the two ends of the line 'replace' the original inductor.

An advantage of this method of representing reactive elements is that they appear to the program to be merely additional transmission lines. This implies that quite complicated terminating networks may be readily simulated. An example is shown in fig. 4.19 and the corresponding 'view' of the network as interpreted by the program

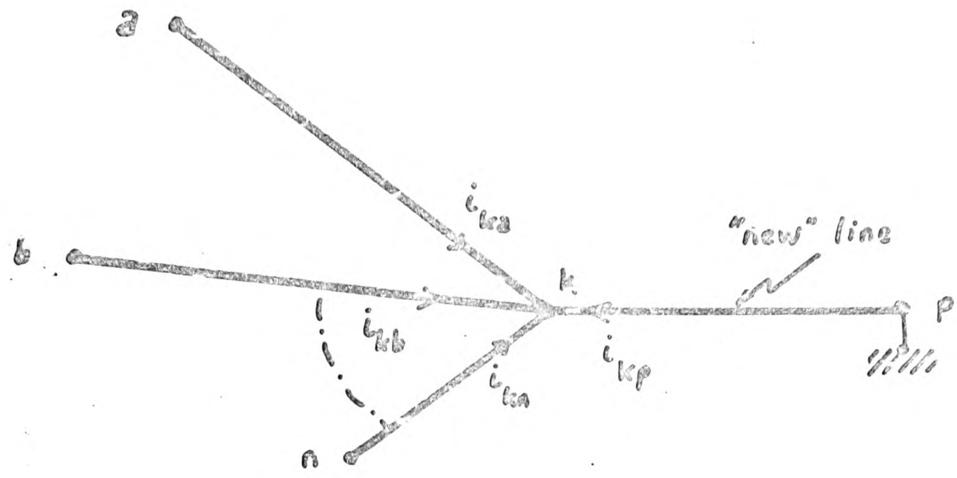


FIG.4.17.

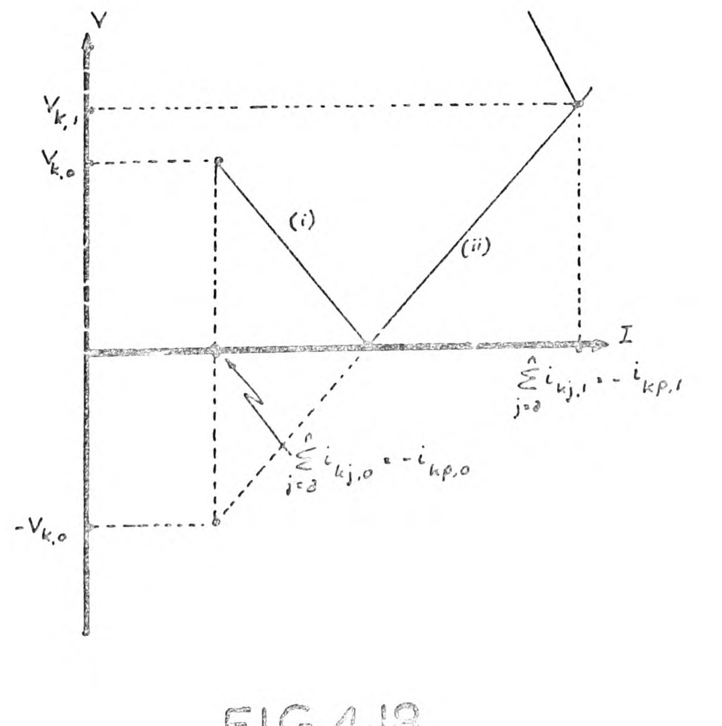


FIG.4.18.

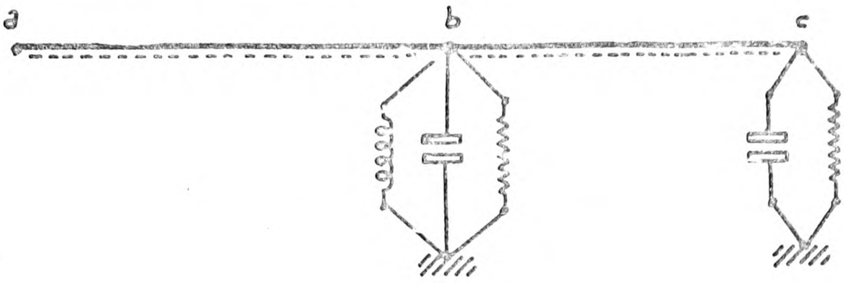


FIG.4.19.

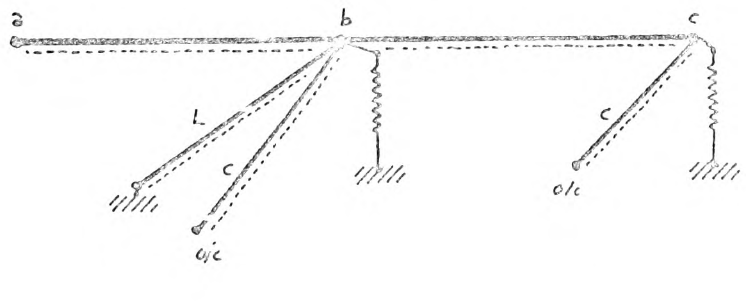


FIG.4.20.

in fig. 4.20.

4.5.2 Accuracy of representation

This idea of representing a lumped reactive element by a distributed constant transmission line necessarily entails a degree of error. The error is inherrent in the assumption that, for example, the voltage across an inductor subject to a changing current varies linearly in a given time interval. This assumption would only be entirely true in the unlikely event of the current changing at a constant rate. An alternative, and in some respects a more meaningful way to view the approximation, is to appreciate that a lumped element, defined as having zero transit time, is being represented by an element whose transit time is finite.

Obviously the smaller j' is made, then the lower is the error content of the result. It is convenient in the program to make j'/2 equal to one time unit thus the actual value assigned to the interval in question is very largely dependent on the topology of the network under examination and the manner in which the program handles it. Quantising the error in absolute terms is thus difficult in general and not particularly meaningful. Dommel[38] has reported that this technique of stub line approximation is less accurate than an alternative which he proposes, that of using trapezoidal integration. This is apparently especially true if more exotic processes such as Richardson extrapolation are used.

The simplicity of stub approximation, and the flexibility which this technique offers more than offset any advantages to be gained by alternative techniques. Certainly the computer storage requirement and program execution time are less where this stub representation method is used.

Experience remains one of the best tools to determine whether or not the error produced in a result is excessive. In general, results show that satisfactory accuracy is achieved if the transit time of the shortest transmission line in the network exceeds the transit time of the line used to represent the reactive element by a factor of at least 4 or 5.

4.6 Series Elements

The program is capable of handling a number of series elements connected between sections of transmission line, or, more generally connected between nodes to which numbers of lines are connected as shown in fig. 4.21. In particular the basic program has facilities for handling series linear resistors, series capacitors and sets of series connected inductors with mutual coupling. The analysis for each of these elements is basically the same and so only that for the series capacitor will be considered in any detail. It will be recalled that series inductors may be represented simply by 'equivalent' transmission lines as described earlier.

Consider a capacitor (C), connected between two nodes a and b. These nodes are, in turn, connected to nodes w, x, y and z via surge impedances of Z_{aw}, A_{ax} etc. as shown in fig. 4.22.

The transit times of all the lines connected to the capacitor equal one time unit.

Across the capacitor ab we have:-

$$i = C.dv/dt \quad .... \quad 4.30$$

Using finite differences and assuming the voltage across C to change linearly in the interval j' (=dt), then we have:-

$$(i_{a,o} + i_{a,1})/2 = (C/j').((V_{a,1} - V_{b,1})-(V_{a,0} - V_{b,0})) \quad .... \quad 4.31$$

where $i_{a,0} = i_{aw,0} + i_{ax,0}$ etc.

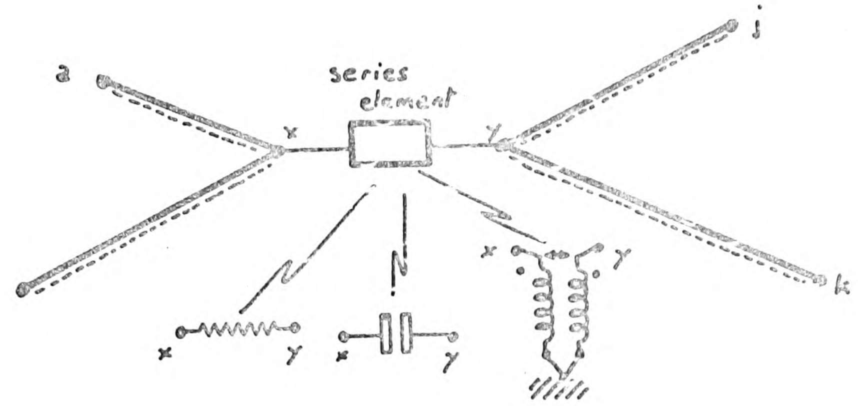


FIG.4.21.

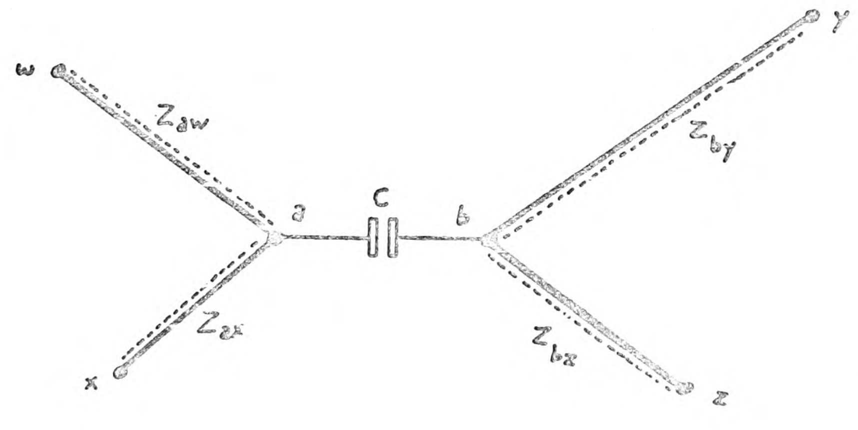


FIG.4.22.

Also we have for the currents at b:-

$$-(i_{b,0} + i_{b,1})/2 = (C/j').((V_{a,1} - V_{b,1})-(V_{a,0} - V_{b,0})) \quad ....4.32$$

where current subscripts have their usual significance.

The normal equations of the graphical method apply to observers on the transmission lines. Hence we have, for example, for line wa:-

$$V_{a,1} + i_{aw,1}.Z_{aw} = V_{w,0} - i_{wa,0}.Z_{aw}$$

Similar equations exist for the other lines. Equation 4.31 may be rearranged as:-

$$V_{a,1}-V_{b,1} -(j'/2C).i_{a,1} = V_{a,0}-V_{b,0} + (j'/2C).i_{a,0} \quad .... 4.33$$

and equation 4.32 becomes:-

$$V_{b,1}-V_{a,1} - (j'/2C).i_{b,1} = V_{b,0}-V_{a,0} + (j'/2C).i_{b,0} \quad .... 4.34$$

Equations 4.33 and 4.34 may be combined with the equations for the observers on the transmission lines in matrix form as:-

$$\begin{bmatrix} Z_{wa} & 0 & 0 & 0 & 1 & 0 \\ 0 & Z_{xa} & 0 & 0 & 1 & 0 \\ 0 & 0 & Z_{yb} & 0 & 0 & 1 \\ 0 & 0 & 0 & Z_{zb} & 0 & 1 \\ -j'/2C & -j'/2C & 0 & 0 & 1 & -1 \\ 0 & 0 & -j'/2C & -j'/2C & -1 & 1 \end{bmatrix} . \begin{bmatrix} i_{aw,1} \\ i_{ax,1} \\ i_{by,1} \\ i_{bz,1} \\ V_{a,1} \\ V_{b,1} \end{bmatrix} = \begin{bmatrix} W_0 \\ X_0 \\ Y_0 \\ Z_0 \\ V_{a,0} - V_{b,0} + j'/2C \; i_{a,0} \\ V_{b,0} - V_{a,0} + j'/2C \; i_{b,0} \end{bmatrix} \quad .... 4.35$$

The terms W_0, X_0 etc. are as defined previously where:-

$$W_0 = V_{w,0} - i_{wa,0}.Z_{aw} \qquad \text{etc.}$$

The coefficient matrix may be pre-computed, and its inverse formed and stored since it does not contain time dependent terms. Such a computation can, however, only be made after all line sectioning has taken place so that the time increment (j') corresponding to the standard time between nodes, has been established.

The size of the coefficient matrix in equation 4.35 will, of course, depend on the number of lines connected to the capacitor. In general it will be square and of order (n+2), where n is the number of such lines. Though there is theoretically no limit to the number of connections which could be made to the capacitor nodes, it has been found necessary in the versions of SUSAN implemented thus far to restrict the number to 4 in the interests of core storage economy.

Other series elements are handled in a similar manner except that the equations corresponding to equation 4.35 appear somewhat altered. An example would be the case of coupled coils which is illustrated in fig. 4.23. Here two coils having self inductances of La and Lb are coupled together via a mutual inductance M as shown. The coupling coefficient, K, is given as usual by:-

$$K = M/La.Lb)^{\frac{1}{2}}$$

The equations for solution become:-

$$\begin{bmatrix} Z_{wa} & 0 & 0 & 0 & 1 & 0 \\ 0 & Z_{xa} & 0 & 0 & 1 & 0 \\ 0 & 0 & Z_{yb} & 0 & 0 & 1 \\ 0 & 0 & 0 & Z_{zb} & 0 & 1 \\ -\frac{L_a}{j'} & -\frac{L_a}{j'} & -\frac{M}{j'} & -\frac{M}{j'} & \frac{1}{2} & 0 \\ -\frac{M}{j'} & -\frac{M}{j'} & -\frac{L_b}{j'} & -\frac{L_b}{j'} & 0 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} i_{aw,1} \\ i_{ax,1} \\ i_{by,1} \\ i_{bz,1} \\ V_{a,1} \\ V_{b,1} \end{bmatrix} = \begin{bmatrix} W_0 \\ X_0 \\ Y_0 \\ Z_0 \\ -\left(\frac{V_{a,0}}{2} + \frac{L_a}{j'} i_{a,0} + \frac{M}{j'} i_{b,0}\right) \\ -\left(\frac{V_{b,0}}{2} + \frac{L_b}{j'} i_{b,0} + \frac{M}{j'} i_{a,0}\right) \end{bmatrix} \quad \ldots. \; 4.36$$

and the solution is effected by inversion of the coefficient matrix as in the previous case.

A further significant item which has yet to be considered is the question of mutual coupling between transmission lines.

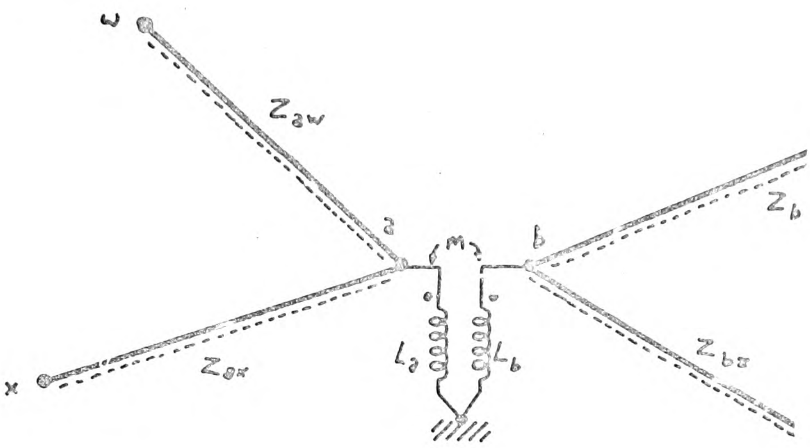


FIG.4.23.

Since this is an important and to some extent independent aspect of the work, it will be treated in a separate later chapter.

The following chapter is concerned with some aspects of the design and implementation of the program itself.

5. PROGRAM FEATURES

5.1. General Considerations

It is probably true to say that a complex computer program has much in common with a literary work, or even with a musical composition. A programer develops a certain 'style' of his own which becomes easily recognisable. In consequence of this fact 'routine' operations - sorting, matrix manipulation etc. - will be handled in different ways by different workers.

Only in the case of the simplest programs is there any real advantage in describing the program logic and operation 'statement by statement'. By the same token very detailed flowcharts, though valuable to those who have to service and possibly modify the program, may only confuse if they are presented as a means of describing the program functions. It is often as informative to 'read' the source program directly rather than to attempt to follow flowcharts and similar 'aids'.

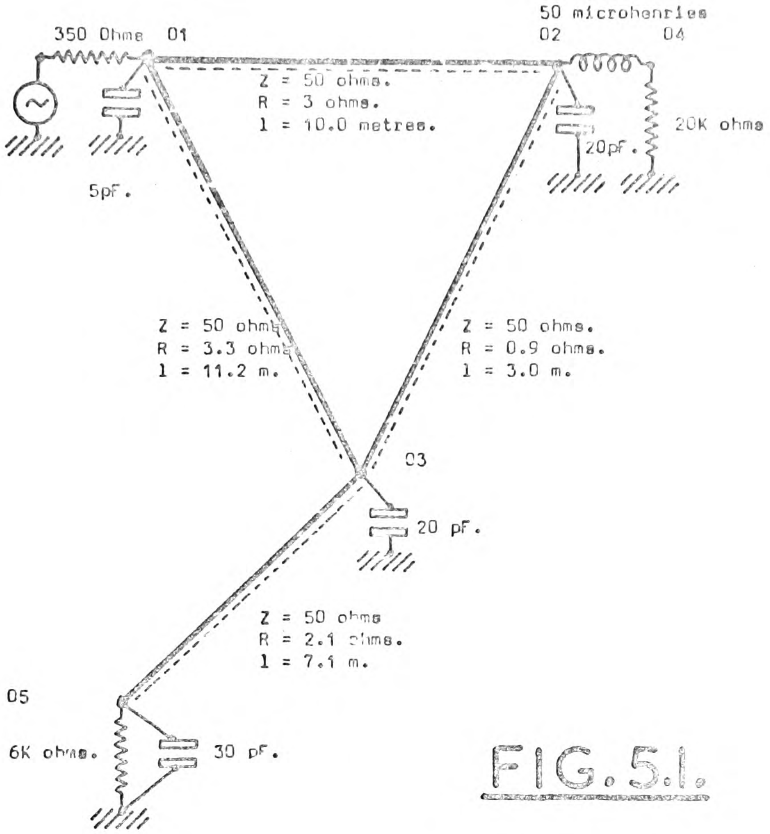
In the present case, therefore, the overall structure of the routine will be described with special emphasis on some of those features which are of particular interest. A complete listing of the whole program together with some of the supporting software is provided in Appendix 1.

The routine is written in Fortran IV and has been implemented on an IBM 1130 computer whose configuration includes card reader/ punch, line printer and disc stores. Since the machine used had only 16K words of fast store (16 bit words), the program was necessarily highly segmented consisting of a main calling routine and a large number of subroutines. During execution the majority of these subroutines are maintained in disc working storage, only being loaded into fast storage - and overlaying existing subroutines - when required. By careful design of the subroutines it is possible to minimise the adverse effect on execution time which this type of technique necessarily produces. Further information on the computer and the Monitor system under which SUSAN operates is given in the manufacturer's publications [40]. It has already been mentioned that an early version of SUSAN is resident in the Institution of Electrical Engineers computer program library and as such is available for general use on payment of a handling charge. At least one attempt is currently being made to implement the routine on an I.C.L. 1905A machine (University of London computer centre).

Certain of the features of SUSAN are best illustrated using examples as follows :-

5.2. Data Input

The data input routines have been designed to make the program as easy as possible to use. Simplicity of operation has been equated to a minimum of data preparation on the part of the user. It is merely necessary only to number the nodes in the network and to supply information on the elements connected at these nodes and on the nature of the transmission lines of other elements joining them. Consider the hypothetical network shown in fig. 5.1. This has been devised to include a variety of circuit elements and configurations. Table 5.i gives the complete set of data necessary to describe this network to SUSAN. The nodes in fig. 5.1. have already been numbered from 1 to 5 and this numbering may be done in any order but it must be continuous - i.e. there must be no missing nodes. The final network, as processed by the computer, will have many more nodes than five since it will be necessary to subdivide each of the transmission lines in order to satisfy the requirement that there must be an equal transit time between adjacent nodes. This process is, however, completely automatic.



350 Ohms
01
50 microhenries
02
04
Z = 50 ohms.
R = 3 ohms.
1 = 10.0 metres.
20K ohms
20pF.
5pF.
Z = 50 ohms
R = 3.3 ohms
1 = 11.2 m.
Z = 50 ohms.
R = 0.9 ohms.
1 = 3.0 m.
03
20 pF.
Z = 50 ohms
R = 2.1 ohms.
1 = 7.1 m.
05
6K ohms.
30 pF.

FIG.5.1.

Table 5.i

Reference number 109
Day 17 Month 08 Year 71
Comment ILLUSTRATION OF DATA INPUT

Voltage multiplier 1.000
Time multiplier 0.001
Output node numbers 1 2 3 4 5
Sectioning tolerance (per-cent) 001
Study duration (microsecs / time multiplier) 200

Node data

Node no.	Node name	Node type	A	B	C	D	E	Init.volts
01	ALPH	2	3	10	500	75	3.5	0
02	BETA	1	0	0	0	0	0	0
03	GAMM	1	0	0	0	0	0	0
04	DELT	3	0	0	0	0	200	0
05	EPSI	3	0	0	0	0	60	0
								ENDING

Connection data

Term.node	Term.node	Z(surge)	R(ohms)	length (m)	vel.fact(%)	L
01	02	50.0	3.0	10.0	60	
01	03	50.0	3.3	11.2	60	
02	03	50.0	0.9	3.0	60	
02	04					0.000050
03	05	50.0	2.1	7.1	60	
						ENDING

Shunt reactor data

Node no.	C(microfarads)	L(henries)
01	0.000005	
02	0.000020	
03	0.000020	
05	0.000030	

ENDING
NO TIME MODS

Data is arranged in four basic classifications. Referring to table 5.1 we have:-

5.2.1 <u>General data</u>

This consists of a reference number, date and alphameric comment for run identification. The use of a voltage multiplier allows the program to handle very large and very small values equally well. Voltages are expressed in volts x voltage multiplier. Times are expressed in microseconds x time multiplier and this time multiplier is selected bearing in mind the order of magnitude of time in a particular problem. In the present case time is described in microseconds x 0.001 = nanoseconds.

The output node numbers, as their name implies, are the numbers of the nodes at which the transient is to be monitored. Up to eight such nodes may be specified in any particular run.

The next item of data presented to the program is the sectioning tolerance. This requires some explanation. In most practical networks handled by SUSAN, the highest common factor of all the transmission line transit times is likely to be very small. An attempt by the program to insert the correct number of additional nodes and connections to produce an inter-node transit time equal to this highest common factor will often fail due to lack of sufficient storage area. In this event the program will auotmatically modify the network, increasing the lengths of some lines, shortening others. The sectioning tolerance is a measure of the amount of length change (in per-cent of the original length) which the user is prepared to tolerate.

In the event that SUSAN is unable to section within the user-specified tolerance, it will increase this tolerance and try again until the best possible sectioning performance, bearing in mind the core store available, has been achieved.

5.2.2 Node Data

Each node in the original network is identified by a number and, optionally, by a name. Each is designated by the user as being of a specific type, each type having a different code number. This classification is determined by the nature of the elements connected to the node so that, for example, a simple node having only transmission lines connected to it would be type '1'. A full list of node types is given in Appendix 2.

The parameter values associated with the node ('A' through 'F' in table 5.1) have different interpretations depending on the node code. Taking node 01 as an example, reference to fig. 5.1 shows that this has a sinusoidal source, capacitor and transmission lines connected to it. The lines and capacitor are handled under the headings 'connection data' and 'shunt reactor data' and the parameter values here thus refer to the source itself.

Thus the 'A' parameter represents, in this case, a code implying that the forcing function is sinusoidal, B represents the peak voltage (10 volts), C is the frequency (hundred of Hz.), D is the initial phase angle of the source at time zero (75 degrees) and E is the source impedance (hundreds of ohms).

The possibility of specifying starting voltages other than zero is catered for by the 'initial volts' column in the node data. In the present case all the 'initial voltages' are zero implying no stored energy in the network before time zero.

5.2.3 Connection Data

Connections between nodes are made mainly by transmission lines. It is only necessary to specify the line surge impedance, resistance, length and velocity factor (as a percentage of the velocity of light), together with information on the nodes which the line connects.

The value assigned to the line resistance has to be something of a compromise. At high frequencies skin effect may become significant and thus the value chosen will usually be somewhat in excess of the simple direct current resistance of the connection. In practice the effect on line resistance on network transients is often marginal and thus a certain degree of latitude is allowable in the choice of resistance values.

Series inductors are denoted by an entry in a special column of the data. In our example such an inductor is connected between nodes 02 and 04 and the relevant entry is given in table 5.1. Series resistors and other components are treated as examples of special nodes and the way in which these are handled is evident from Appendix 2.

5.2.3 Shunt Reactor Data

Shunt reactors are taken as a generic term for capacitors and inductors connected in shunt at any node in the network. Handling these separately from the node or connection data makes for considerable flexibility in the variety and complexity of the networks which can be analysed by the system. Any combination of capacitor or inductor or both together may be specified as connected at any node. These reactive elements will be subsequently replaced by short lengths of open or short circuited transmission line of appropriate surge impedance in the manner described in a preceeding chapter. In the example which we are using there are no shunt inductors but four shunt capacitors have been included. The ability to simulate such capacitors is a valuable one since it means that, for example, circuit stray capacitances and the like may be readily handled.

The remainder of the data is concerned with timed node modifications of which there are none in this instance. The facility exists to modify the entry for any node at any pre-selected time after time zero. Furthermore any number of such modifications can be made either at the same time or at intervals throughout the run. One of the more important uses of this facility is to simulate non-standard forcing functions - the generation of a pulse train would be an example.

5.3 System Messages

SUSAN produces a variety of coded messages for the benefit of the user during the course of a run. The complete set of messages is listed in Appendix 3 and a large number of these are concerned with reporting on the results of the many error detection tests which are performed on the input data.

Generally messages are of three types. One set gives information on logical decisions taken by the routine or requests operator intervention. Another set gives the results of error detection tests where 'fatal' errors - those which prohibit further program execution - have been detected. An example might be the detection of a connection made to a node which has not been specified as being a part of the network. The third set reports on 'possible' data errors - situations where the input data appears to be unusual but not capable of misinterpretation.

Messages appear either on the output lineprinter or on the machine operators' console typewriter if intended for him. A certain amount of operator intervention, involving setting console program switches etc. is sometimes required. The great flexibility of the output arrangements necessitate this intervention - indeed the program can be made to operate almost in an 'interactive' mode with the user if necessary. On the other hand it may be argued that operator intervention should be avoided - especially if the routine is being used in the absence of the user. In such cases provision is made for the program to be run with 'standard' or 'default' options, thus eliminating the necessity for any intervention.

5.4 <u>Output Facilities</u>

As a general rule transients seem to be most easily interpreted if presented in a graphical form. Usually programs such as this produce output in the form of a list of voltages against time for a number of equally spaced times throughout the run. The preparation of a graph is then a manual process. Automatic graph plotters can, of course, be used if the computer happens to be equipped with this type of equipment.

SUSAN provides an extremely flexible output system which is capable of providing output in any one of four different ways or as a combination of these if required. It offers the choice of :-

i. Output in the form of a list as described above.

ii. A 'graphical' output where the lineprinter is used as a form of graph plotter. An added feature in this case is that these graphs may be 'compressed' in the sense that only those sections in which there is a variation in voltage are printed.

iii. Output to disc file and subsequently to card output.

iv. True graphical output. The machine used for the studies (I.B.M. 1130) was not equipped with a graph plotter of the conventional kind. It has, however, an interface to a Solartron HS7 analog machine equipped with digital to analog converters. Data previously stored on disc files (output iii above) can be converted to graphical form using a standard X-Y plotter as the output device. A program for performing this function forms a part of the SUSAN system.

Examples of each of these forms of output are given in the following chapter which is concerned with applications of the system.

5.5 <u>Line Sectioning</u>

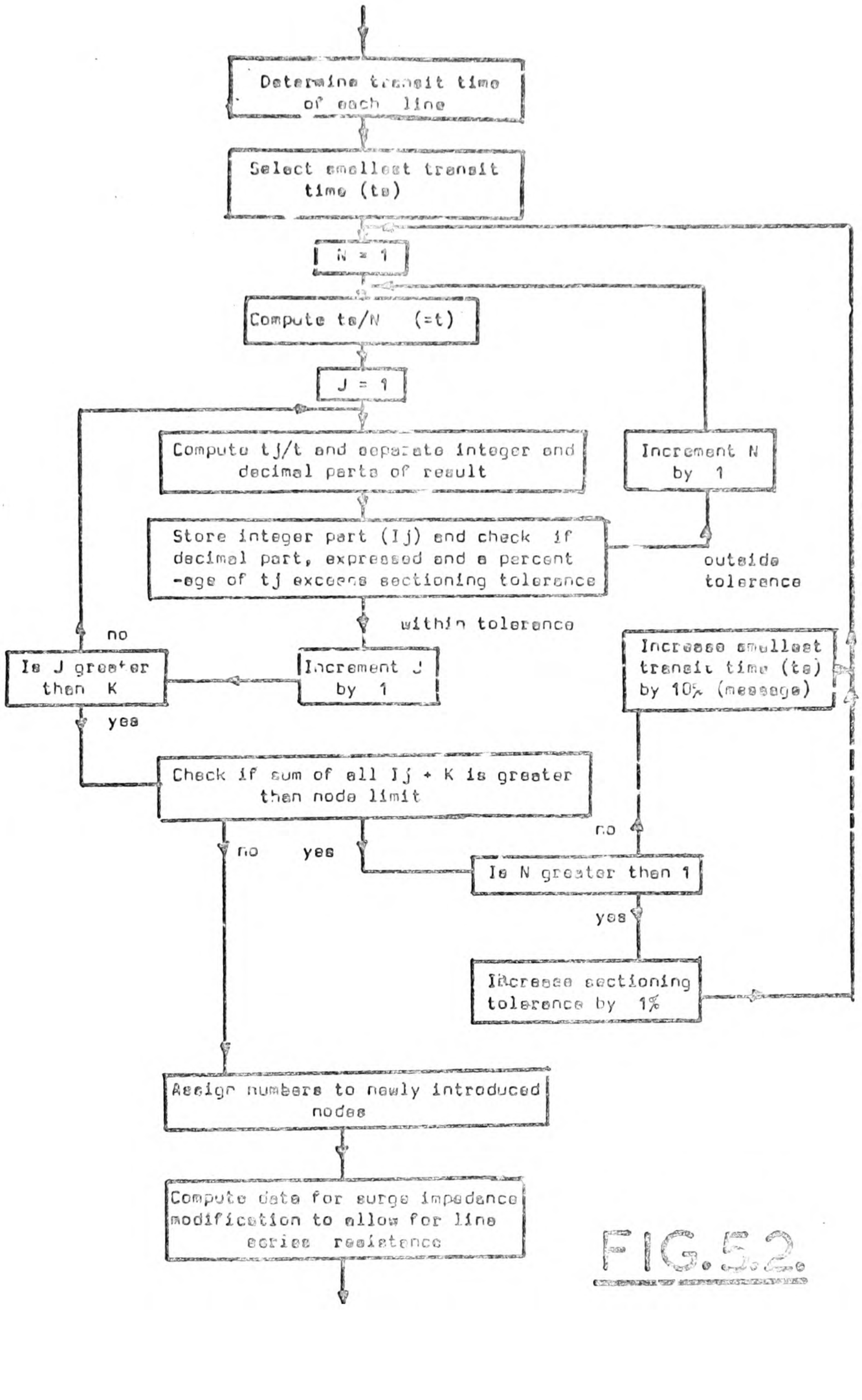
The necessity to divide transmission lines into a number of sections has already been mentioned on several occasions. In most practical cases this operation cannot be done exactly unless a great deal of fast storage is available since the number of nodes which would have to be introduced might become very large.

The program as implemented on the I.B.M. 1130 computer can accomodate up to 100 nodes of all types. This number is a function only of the amount of starage available in the machine and does not represent a limitation on the program. Fig. 5.2 shows, in simplified form, the logic necessary to perform the line sectioning operation. Basically the procedure is a trial and error one. Having evaluated the transit times of all the transmission lines in the network - from considerations of their lengths and velocity factors - the smallest of these is selected.

This is then divided into each of the other transit times in turn and the result separated into an integer and a decimal part. The decimal part is discarded and thus the line in question is effectively 'shortened' by that amount. This operation represents an error and this error is quantified by expressing the decimal part as a percentage of the original line transit time. If the resulting percentage is greater than the user-specified 'sectioning tolerance' - typically 1% - then the operation is deemed to have been a failure. The smallest transit time is then reduced and the process repeated until all of the errors are within the specified tolerance.

It may well prove necessary for the routine to introduce more than the maximum number of 100 nodes permitted into the network to meet the tolerance criterion. This is obviously not possible and in such an event the routine will automatically reduce the sectioning tolerance and try again. It is then necessary for the user to decide whether the resulting computation is sufficiently accurate for his purposes.

In some networks the differential between the smallest and largest transit times may be very great. An example might be a circuit consisting of long power transmission lines together with very short lengths of cable introduced here and there. In such a case there is a danger that the initial division of the smallest transit time into the others will result in



Determine transit time of each line
Select smallest transit time (ts)
N = 1
Compute ts/N (=t)
J = 1
Compute tj/t and separate integer and decimal parts of result
Increment N by 1
Store integer part (Ij) and check if decimal part, expressed and a percent -age of tj exceeds sectioning tolerance
outside tolerance
within tolerance
no
Is J greater than K
Increment J by 1
Increase smallest transit time (ts) by 10% (message)
yes
Check if sum of all Ij + K is greater than node limit
no
no
yes
Is N greater than 1
yes
Increase sectioning tolerance by 1%
Assign numbers to newly introduced nodes
Compute data for surge impedance modification to allow for line series resistance

FIG.5.2.

the introduction of too many nodes into the network. Here again the program exhibits an automatic response and it will increase the length of the shortest line by a fixed percentage and attempt the entire procedure again.

The SUSAN system has now been used on a very wide variety of networks and in almost every case the sectioning routine has been able to produce adequate results. Though not usually necessary it may be helpful to the user to have available a 'picture' of the network after sectioning and after the newly introduced nodes have been numbered. The program has this facility if required.

5.6 Topological Representation

The speed of execution of programs such as this is very largely dependent on the ease with which the nodes connected to a particular node can be identified. The computation of voltage at any node requires two basic sets of information. Firstly the voltages at each of the nodes to which the node in question is immediately connected must be known one time unit in the past, and secondly the routine must know the surge impedances of each of the connecting transmission lines together with any additional data which might be required - surge impedance modification information to allow for line resistance, node type data etc.

The simplest method of approach is to use a square matrix whose order corresponds to the number of nodes in the network. The intersection of a row and column whose coordinates correspond to two of the network nodes may then represent the parameters of the connection joining them.

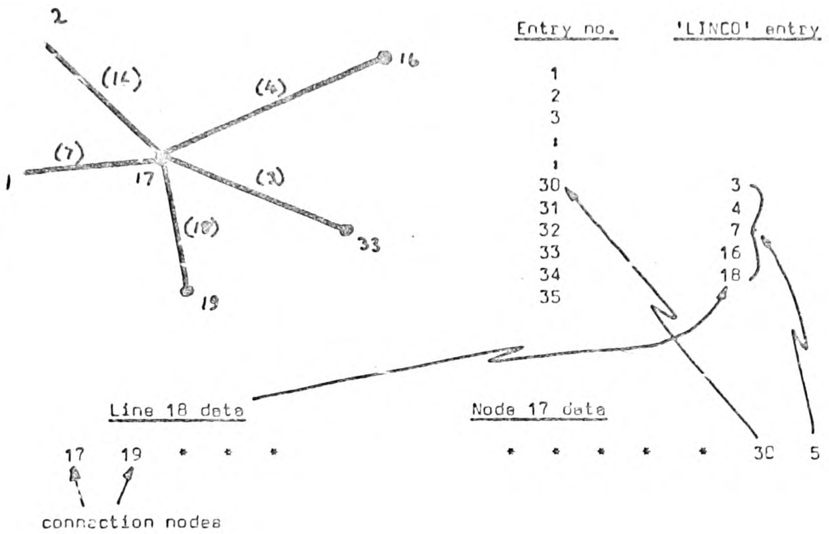
Though simple this technique is extremely wasteful of storage area since the core size required increases as the square of the number of network nodes. It does, of course, provide the facility of being able to specify connections between each node and every other (e.g. 100 connections in the case of a network with 10 nodes), though this advantage has little practical value.

A rather more practical and economical system is used in the program. This places a restriction on the number of lines which can be accomodated in any network - typically slightly more than the total maximum number of nodes - and as a result great savings in core requirements are effected whilst retaining the necessary speed of identification.

The system employed is illustrated in fig. 5.3. A 'double addressing' process is utilised, the heart of which is a 'line connection' (LINCO) table. This is a single column vector having twice as many entries as are lines within the network (after line sectioning). Fig. 5.3 shows the relevant entries for node 17 of a hypothetical network. This node is connected to nodes 1, 2, 16, 33 and 19, via lines (7), (16), (4), (3) and (18) as shown. It should be noted that the line numbers are assigned automatically by the routine and remain unknown to the user.

Node 17 data contains two entries relevant to its position within the network. The first of these (30 in fig. 5.3) is a pointer to an address in LINCO. whilst the second is a count. The actual LINCO entries themselves are line numbers and thus the pointer directs the routine's attention to entry 30 in LINCO which is the number of the first line ((3)) connected to the node. The count of 5 implies that the remaining line numbers - of which there are 4 - may be identified by the next 4 entries in LINCO (entries 31 to 34 respectively). Having thus identified the lines connected to the node in question, the numbers of the nodes at the remote ends of these lines are determined by reference to the line data itself. Reference to fig. 5.3. shows part of the data for line 18 which consists of the numbers of the two terminating nodes.

This technique has been successfully used by the writer for other programs where this topological problem exists - typically power system



Total number of 'LINCO' entries = 2 x number of lines included in network

(7) = line number (assigned by program)

33 = node number (assigned by user)

FIG. 5.3.

load flows and similar types of computation. It has proved to be both simple and effective and relatively fast since only a small amount of arithmetic has to be performed.

5.7 'Past History' Analysis

This process effectively increases the speed of operation of the program by a considerable amount. In fact, depending on the nature of the circuit under investigation, the use of past history analysis has resulted in execution times being at least halved and in some cases reduced by factors of three or four.

The idea behind the technique is a simple one. Instead of simply computing the voltage and currents at each node for every time increment in the normal way, use is made of the fact that the electrical conditions at many nodes within any network - especially those contained as additional nodes within long transmission lines - will only alter infrequently as voltage and current waves pass through them. This is especially true if the system is subject to step function excitation.

Two values of each voltage and current throughout the network are in storage at any one time. At the end of a scan through the complete set of nodes information is available on the voltages and currents at the present time, together with similar information one time unit in the past. In the normal way the 'old' historical information is discarded and the newly computed data becomes historical data for the next pass. In the case where 'past history analysis' is being employed a comparison is made between every 'old' item of data and every corresponding 'new' item. A simple matrix containing only 0 and 1 terms is used to register coincidence or difference.

During the following pass the nodes connected to each node under examination are identified in the normal way. A check on the 'coincidence' matrix then rapidly determines whether or not the voltages and currents at each of these nodes changed during the time interval prior to the current one. If the results of this series of tests indicate that the conditions at all relevant points surrounding the node being considered were the same at the commencement of the current time interval as they were one time unit before, then it follows that the new values of voltage and current do not, in fact, need to be calculated since they will be identical to those already in storage. In physical terms this means that no waves have reached the node during the time of interest.

Naturally the additional complexity of making these coincidence tests results in an increase in execution time. There is also some penalty incurred in respect of the amount of core storage required to accomodate the routine. In practice, however, the savings in time which are obtained by being able to simply copy some results rather than have to compute them more than offset the increases and the nett result is a considerable improvement in operating efficiency. The core storage penalty is not excessive, amounting to only approximately (2.l + n) words of fast store where 'l' is the number of line sections used and n is the number of network nodes.

6. SAMPLE STUDIES

6.1 Choice of examples

The program is intended to be useful in a wide range of areas within the whole electrical engineering field. This intention is reflected in the general form of the input and output arrangements which were described in the previous chapter.

The examples chosen to illustrate the operation of the routine are therefore taken from a variety of fields. Some are hypothetical in nature and are primarily designed to illustrate particular program features, whilst others relate more closely to practical transient problems. In order to validate the results produced by SUSAN a considerable amount of laboratory work has been carried out using time domain reflectometer equipment and the agreement between laboratory and computed results so obtained is illustrated.

A final example, not included in this chapter, illustrates the capability of the program to handle networks in which there is mutual interaction between transmission lines or between phases of a multiphase line. This has been included in the following chapter where the whole question of mutual line coupling is considered in more detail.

6.2 The 'Data Input' example

The network shown in fig. 5.1 has already been used to illustrate the form of the data input to the program. A logical conclusion to this is example is thus to run the data (as given in table 5.1) and note the results which are produced. Such an example as this also affords a useful opportunity to study the various forms of output which are available.

Fig. 6.1 is a reproduction of the initial output produced at the commencement of the run. This is basically an echo check of the input data together with a series of messages giving diagnostic and other information.

SURGE SYSTEM ANALYSIS PROGRAM

(SUSAN - 1)

REFERENCE NUMBER IS 109

DATE 17/ 8/ 71

COMMENT - ILLUSTRATION OF DATA INPUT

VOLTAGES ARE IN VOLTS X 1.000
TIME IS IN MICROSECS X 0.001

NODE DATA (BEFORE SECTIONING)

NO.	NAME	TYPE	A	B	C	D	E	INITIAL VOLTS
1	ALPH	2	3.000	10.000	500.000	75.000	3.500	0.0
2	BETA	1						0.0
3	GAMM	1						0.0
4	DELT	3	0.000	0.000	0.000	0.000	200.000	0.0
5	EPSI	3	0.000	0.000	0.000	0.000	60.000	0.0

MESSAGE (CODE=S10) - END OF NODE DATA - NO FATAL ERRORS DETECTED

CONNECTION DATA

NODE	TO NODE	Z(SURGE) (OHMS)	RESISTANCE (OHMS)	LENGTH (METRES)	PROP.VEL. (PC.OF C)
1	2	50.0	3.0	10.0	60.0
1	3	50.0	3.3	11.2	60.0
2	3	50.0	0.9	3.0	60.0
2	4	SERIES INDUCTOR OF 0.000050 HENRIES			
3	5	50.0	2.1	7.1	60.0

MESSAGE (CODE=S11) - NO DETECTABLE ERRORS IN CONNECTION DATA

FIG 6.1

SHUNT REACTIVE ELEMENTS

AT NODE	C (MICROFARADS)	L (HENRIES)
1	0.000005	
2	0.000020	
3	0.000020	
5	0.000030	

MESSAGE (CODE=S26) - END OF SHUNT REACTOR DATA

MESSAGE (CODE=S29) - NO TIMED NODE MODIFICATIONS REQUESTED

MESSAGE (CODE=S20) - SPECIFIED SECTIONING TOLERANCE IS 1 PER-CENT

MESSAGE (CODE=S13) - 58 ADDITIONAL NODES ADDED FOLLOWING SECTIONING

MESSAGE (CODE=S14) - SECTIONING TOLERANCE INCREASED TO 2 PER-CENT

MESSAGE (CODE=S15) - STUDY DURATION IS 0.199 MICROSECS

Messages S10 and S11 shown on the output indicate that the routine detected no anomalies in the input data. Although the user specified sectioning tolerance was 1%, message S14 indicates that the program was forced - as result of lack of fast store capacity - to increase this tolerance to 2%. 58 additional nodes, excluding those used to represent the shunt capacitors in the network, were added following the sectioning operation.

In some cases it may be desirable to monitor the network topology following line sectioning. Fig. 6.2 shows a portion of the listing which is produced following such a request. This indicates exactly which nodes are connected to each other and also distinguishes between 'original' nodes and the 'additional' nodes added automatically by the program. This facility is primarily intended for use in those cases where a drastic increase in sectioning tolerance has to be made, so much so that the physical differences between the circuit actually computed and the circuit as initially specified may have become considerable.

The problem specification required that all voltages at nodes 1 through 5 should be monitored. A maximum of eight such voltages may be noted during any single run. Fig. 6.3 shows these voltages listed as functions against time. This represents the most 'conventional' of the output forms available though experience has shown that the other 'graphical' output forms provide more acceptable and useful alternatives.

An example of the graphical output in which the line printer is used as the graph plotter is shown in fig. 6.4. Results are quantised into one of one hundred levels, each level being represented by a single printer position. A rounding up and down facility is incorporated into the routine which performs this quantising operation. The limits of maximum and minimum voltage can either be specified by the user - via the console keyboard - or the routine may be used in its 'auto-scaling' mode. Here a search of the entire

CONNECTION DATA AFTER SECTIONING

NODE	IS CONNECTED TO ORIGINAL NODE	OR ADDITIONAL NODE
1		
		6
		25
		64
2		
	4	
		24
		46
		65
3		
		45
		50
		51
		66
4		
	2	
5		
		63
		67
6		
	1	
		7
7		
		6
		8
8		
		7
		9
9		
		8
		10
10		
		9
		11
11		
		10
		12
12		
		11
		13
13		
		12
		14
14		
		13
		15
15		
		14
		16
16		
		15
		17
17		
		16
		18

FIG 6.2

SUSAN 1 - OUTPUT

TIME	(V 1)	(V 2)	(V 3)	(V 4)	(V 5)
0.000	0.617	0.000	0.000	0.000	0.000
2.777	0.617	0.000	0.000	0.000	0.000
5.555	0.666	0.000	0.000	0.000	0.000
8.333	0.666	0.000	0.000	0.000	0.000
11.111	0.625	0.000	0.000	0.000	0.000
13.888	0.625	0.000	0.000	0.000	0.000
16.666	0.669	0.000	0.000	0.000	0.000
19.444	0.669	0.000	0.000	0.000	0.000
22.222	0.633	0.000	0.000	0.000	0.000
25.000	0.633	0.000	0.000	0.000	0.000
27.777	0.670	0.000	0.000	0.000	0.000
30.555	0.670	0.000	0.000	0.000	0.000
33.333	0.640	0.000	0.000	0.000	0.000
36.111	0.640	0.000	0.000	0.000	0.000
38.888	0.672	0.000	0.000	0.000	0.000
41.666	0.672	0.000	0.000	0.000	0.000
44.444	0.647	0.000	0.000	0.000	0.000
47.222	0.647	0.000	0.000	0.000	0.000
50.000	0.674	0.000	0.000	0.000	0.000
52.777	0.674	0.000	0.000	0.000	0.000
55.555	0.653	0.508	0.000	0.000	0.000
58.333	0.653	0.508	0.000	0.534	0.000
61.111	0.677	0.704	0.356	0.534	0.000
63.888	0.677	0.704	0.356	0.713	0.000
66.666	0.659	0.574	0.461	0.713	0.000
69.444	0.659	0.574	0.461	0.566	0.000
72.222	0.679	0.665	0.683	0.566	0.000
74.999	0.679	0.665	0.683	0.671	0.000
77.777	0.665	0.907	0.925	0.671	0.000
80.555	0.665	0.907	0.925	0.920	0.000
83.333	0.682	1.128	0.778	0.920	0.000
86.111	0.682	1.128	0.778	1.139	0.000
88.888	0.670	0.831	0.871	1.139	0.000
91.666	0.670	0.831	0.871	0.815	0.000
94.444	0.685	0.840	0.790	0.815	0.000
97.222	0.685	0.840	0.790	0.842	0.000
99.999	0.675	0.864	0.857	0.842	0.450
102.777	0.675	0.864	0.857	0.865	0.450
105.555	0.689	0.837	0.868	0.865	0.899
108.333	0.689	0.837	0.868	0.835	0.899
111.110	0.600	0.888	0.830	0.835	1.176
113.888	0.600	0.888	0.830	0.891	1.176
116.666	0.732	0.840	0.866	0.891	1.679
119.444	0.732	0.840	0.866	0.837	1.679
122.222	0.454	0.860	0.846	0.837	1.648
124.999	0.454	0.860	0.846	0.861	1.648
127.777	0.517	0.869	0.860	0.861	1.589
130.555	0.517	0.869	0.860	0.869	1.589
133.333	1.024	0.862	0.862	0.869	1.622
136.110	1.024	0.862	0.862	0.862	1.622
138.888	1.365	0.873	0.919	0.862	1.594
141.666	1.365	0.873	0.919	0.874	1.594
144.444	1.059	0.872	1.139	0.874	1.700
147.221	1.059	0.872	1.139	0.872	1.700
149.999	1.044	0.873	1.210	0.872	1.633
152.777	1.044	0.873	1.210	0.873	1.633
155.555	1.089	0.932	1.346	0.873	1.651
158.332	1.089	0.932	1.346	0.935	1.651
161.110	1.033	1.122	1.467	0.935	1.666
163.888	1.033	1.122	1.467	1.131	1.666
166.666	1.165	1.239	1.353	1.131	1.654
169.443	1.165	1.239	1.353	1.245	1.654
172.221	1.022	1.324	1.374	1.245	1.679
174.999	1.022	1.324	1.374	1.328	1.679
177.777	1.121	1.308	1.382	1.328	1.621
180.554	1.121	1.308	1.382	1.307	1.621
183.332	1.069	1.178	1.444	1.307	1.435
186.110	1.069	1.178	1.444	1.171	1.435
188.888	1.108	1.402	1.413	1.171	1.357
191.665	1.108	1.402	1.413	1.414	1.357
194.443	1.089	1.615	1.454	1.414	1.238
197.221	1.089	1.615	1.454	1.626	1.238

FIG 6.3

VOLTAGE AT NODE 3

TIME	VOLTS
0.000	0.000
2.777	0.000
5.555	0.000
8.333	0.000
11.111	0.000
13.888	0.000
16.666	0.000
19.444	0.000
22.222	0.000
25.000	0.000
27.777	0.000
30.555	0.000
33.333	0.000
36.111	0.000
38.888	0.000
41.666	0.000
44.444	0.000
47.222	0.000
50.000	0.000
52.777	0.000
55.555	0.000
58.333	0.000
61.111	0.356
63.888	0.356
66.666	0.461
69.444	0.461
72.222	0.683
74.999	0.683
77.777	0.725
80.555	0.725
83.333	0.778
86.111	0.778
88.888	0.871
91.666	0.871
94.444	0.790
97.222	0.790
99.999	0.857
102.777	0.857
105.555	0.866
108.333	0.866
111.110	0.830
113.888	0.830
116.666	0.866
119.444	0.866
122.222	0.846
124.999	0.846
127.777	0.860
130.555	0.860
133.333	0.862
136.110	0.862
138.888	0.919
141.666	0.919
144.444	1.139
147.221	1.139
149.999	1.210
152.777	1.210
155.555	1.346
158.332	1.346
161.110	1.467
163.888	1.467
166.666	1.353
169.443	1.353
172.221	1.374
174.999	1.374
177.777	1.382
180.554	1.382
183.332	1.444
186.110	1.444
188.888	1.413
191.665	1.413
194.443	1.454
197.221	1.454
199.999	1.572
202.777	1.572
205.554	1.563
208.332	1.563
211.110	1.644
213.888	1.644
216.665	1.689
219.443	1.689
222.221	1.581
224.999	1.581
227.776	1.564
230.554	1.564
233.332	1.534
236.110	1.534
238.887	1.530
241.665	1.530
244.443	1.519
247.221	1.519
249.998	1.516
252.776	1.516
255.554	1.514
258.332	1.514
261.109	1.536
263.887	1.536
266.665	1.536
269.443	1.536
272.220	1.577
274.998	1.577
277.776	1.832
280.554	1.832
283.331	2.073
286.109	2.073
288.887	2.137
291.665	2.137
294.442	2.137
297.270	2.337
299.998	[illegible]

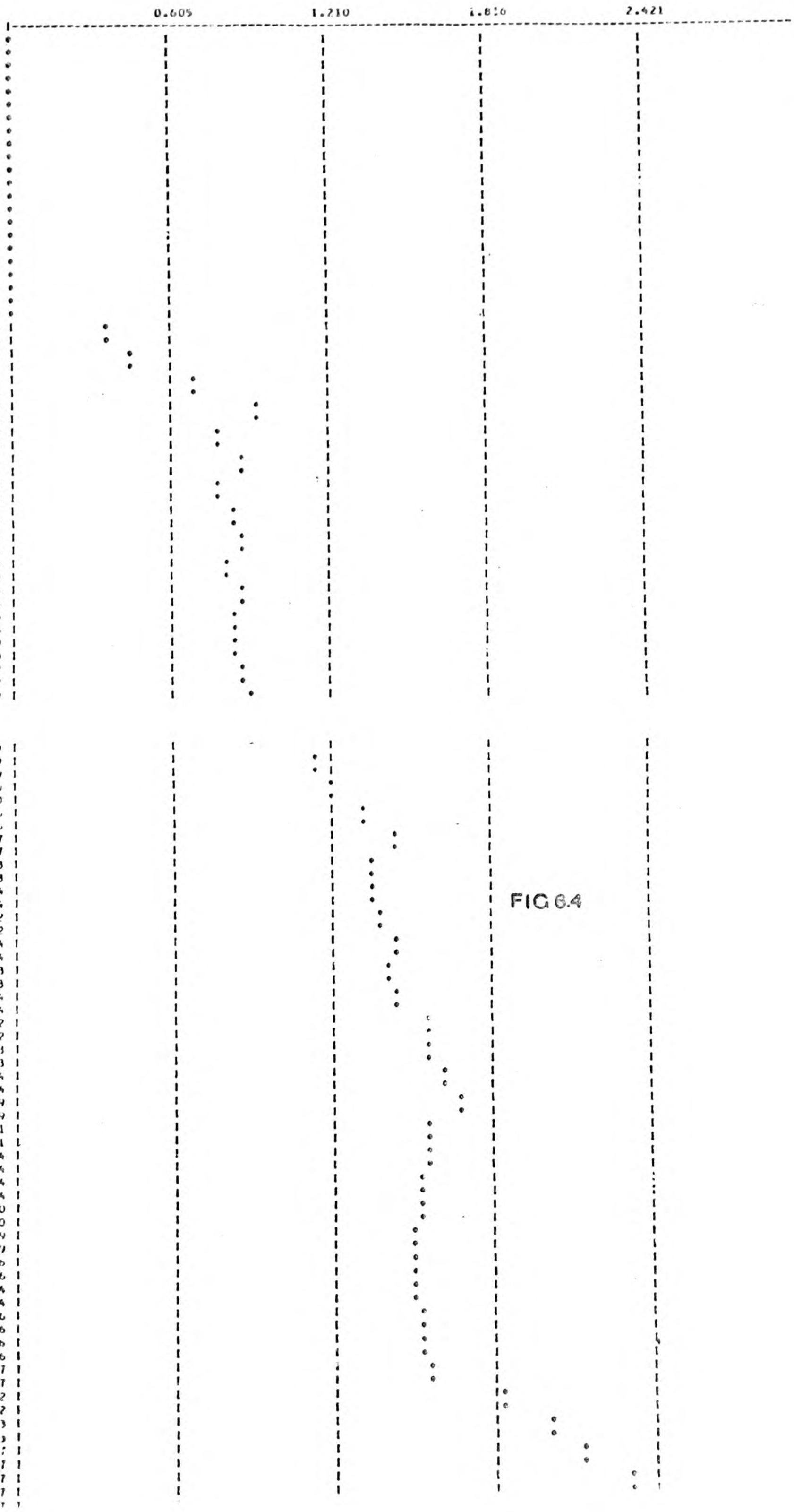


FIG 6.4

set of the results for a given node is made prior to commencing the plotting operation. The largest and smallest results so obtained set the high and low limits for the voltage axis. Each interval of time is represented by a printer line spacing and the actual values of voltage and time are printed to the left of the graph for reference.

A further degree of sophistication is provided by the 'graph truncation' option used with the line printer graphical output. In some instances the voltage at particular nodes may remain constant for long periods of time. This is especially true where the network contains long transmission lines and step excitation is being employed. In such a case the line printer graphs produced may be physically very long and inconvenient. Using 'graph truncation' sections of the graph in which there is no change in voltage level are automatically omitted and consequently the time axis becomes non-linear. This fact implies that a certain amount of care has to be excercised in interpreting the resulting graphs.

Selection of output options etc. is carried out using data input switches in response to requests made from time to time on the console typewriter. In a sense the program may thus be operated in an 'interactive' mode with the user and a sample of a typical console log following a run is shown in fig. 6.5. As has already been mentioned in the previous chapter the program may be run with 'default' options thus obviating the need for user intervention during a run. Default options include the provision of printer graphical output and graph truncation.

Fig. 6.6 illustrates the alternative form of graphical output available with the SUSAN system. Here the computed values of voltage at each interval of time and at each node under consideration are stored on a disc file at the termination of the run. Plotting of the output graphs is then carried out 'off-line' - at least as far as the program is concerned - using the

```
SUSAN - 1

REF. NO. IS 109   DATE  17/ 8/ 71

MESSAGE (CODE=S16)  - SWITCH 1 ON FOR SECTION DATA - PUSH START

MESSAGE (CODE=S17)  - SWITCH 6 ON IF NEW RUN FOLLOWS THIS
                      PUSH START

MESSAGE (CODE=S22)  - SWITCH 2 ON FOR GRAPHICAL OUTPUT, OFF FOR NUMERICAL OUTPUT
                      PUSH START

MESSAGE (CODE=S23)  - GRAPHICAL OUTPUT REQUESTED
                      SWITCH 3 ON FOR AUTO-SCALING, OFF FOR MANUAL
                      PUSH START

MESSAGE (CODE=S27)  - SWITCH 4 ON FOR GRAPH TRUNCATION
                      PUSH START

MESSAGE (CODE=S27)  - SWITCH 4 ON FOR GRAPH TRUNCATION
                      PUSH START

MESSAGE (CODE=S27)  - SWITCH 4 ON FOR GRAPH TRUNCATION
                      PUSH START

MESSAGE (CODE=S27)  - SWITCH 4 ON FOR GRAPH TRUNCATION
                      PUSH START

MESSAGE (CODE=S27)  - SWITCH 4 ON FOR GRAPH TRUNCATION
                      PUSH START

MESSAGE (CODE=S31)  - SWITCH 7 ON IF RESULTS TO BE STORED IN DATA BANK
                      PUSH START

MESSAGE (CODE=S32) - TYPE RESULT NODE NUMBER (XXX) AND FILE NUMBER (YYY)
FORMAT IS XXXYYY (XXX = 0 IF ALL DATA STORED)

 4  1
MESSAGE (CODE=S34)  - NODE    4 RESULTS STORED IN FILE SD 1

MESSAGE (CODE=S32) - TYPE RESULT NODE NUMBER (XXX) AND FILE NUMBER (YYY)
FORMAT IS XXXYYY (XXX = 0 IF ALL DATA STORED)

 3  2
MESSAGE (CODE=S34) - NODE    3 RESULTS STORED IN FILE SD 2
```

FIG 65

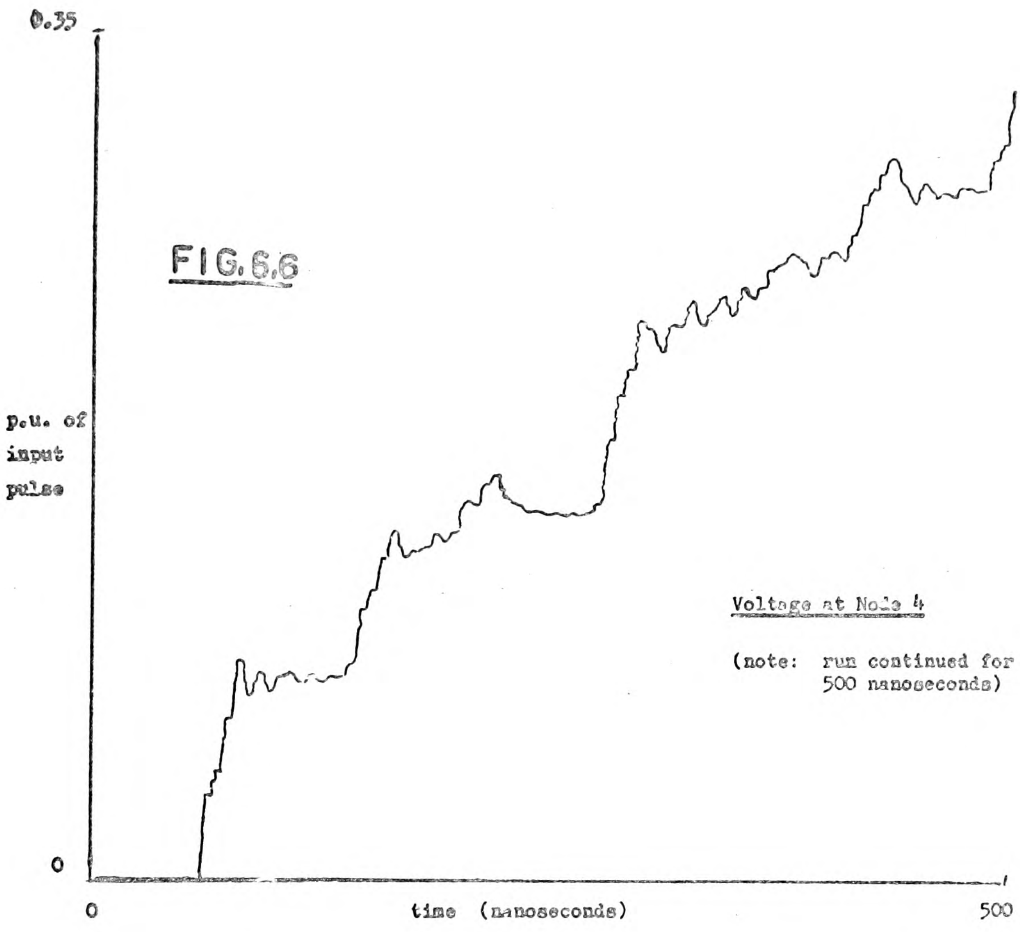


FIG. 6.6
0.35
p.u. of input pulse
0
0
time (nanoseconds)
500
Voltage at Node 4
(note: run continued for 500 nanoseconds)

combination of 1130 digital and Solartron HS7 analog computers.

The two computers are interfaced via an interface unit containing anlog/digital, digital/analog and control hardware. In the present application only the digital/analog converters are required. Results from disc file are quantised to one of 2048 levels and plotted using a conventional X-Y flatbed plotter. Unlike the graphs produced by the lineprinter, these are not plotted 'point by point' but are drawn as continuous lines. The plotting routine thus incorporates facilities for estimating the coordinates of intermediate points between those available from disc resulting in the production of a fairly smooth curve passing through all of the computed points. An additional facility permits graphs to be drawn in 'histogram' form - especially useful when simple systems with step excitation are being studied since the resulting transients often have a stepped form.

The program for this plotting operation is listed in Appendix 1. It may be mentioned that it has also found some use for plotting statistical and other data for the writer's colleagues in addition to its primary role as a part of the SUSAN system.

6.2.1 Line losses

It was mentioned in an earlier chapter that line resistance effects have only limited influence to the form of most transient responses. In general they affect surge voltage amplitudes rather than the shape of the transient response. The present example provides an opportunity to examine the effect of line resistance in a practical case. Fig. 6.7 shows a comparison between transients monitored at the same point in our test network. In one case the line resistances, as detailed in the network data of table 5.1 have been included, whilst in the other case the lines are all assumed to be lossless.

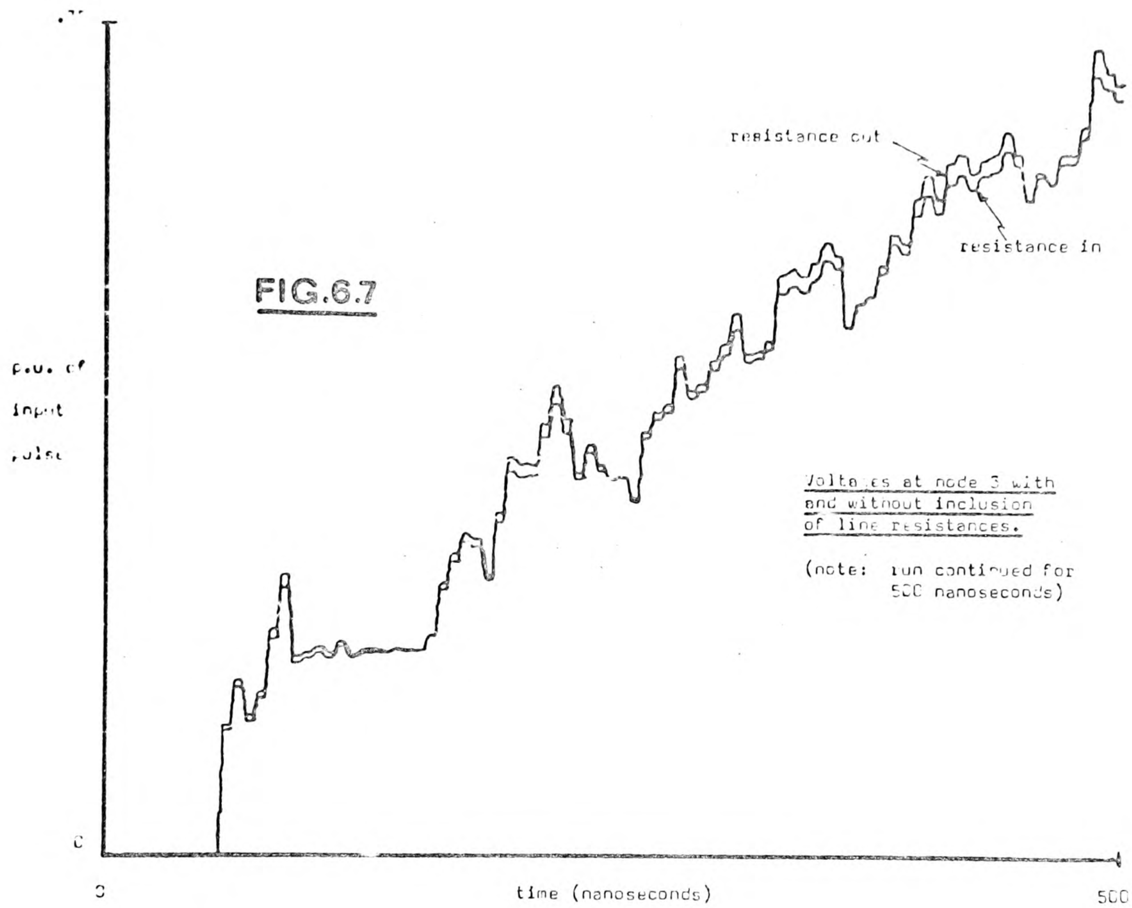


FIG.6.7
p.u. of
input
pulse
0
0
time (nanoseconds)
500
resistance out
resistance in
Voltages at node 3 with
and without inclusion
of line resistances.
(note: run continued for
500 nanoseconds)

6.2.2 Processor time

Figures for the time required to execute a run with this program are possibly not so significant as might be the case with many other programs. Runs using the SUSAN system vary from a few seconds to some hours depending almost entirely on the nature of the network being investigated as well as on other obvious factors such as the number of results required. The processing routines for most of the 'special' node types - voltage sources, coupled coils, series capacitors etc. - are held on disc file to economise on core storage and are only entered into the fast store when called by the controlling routine. Hence a network which contains a number of these special nodes will require a large number of disc to core transfers during the execution phase, all of which serve to prolong the execution time.

However the timing figures for the example just illustrated are presented below as an indication of typical values.

Pre-execution phase (core load preparation, subroutine location etc.)	-	3.5 minutes
Data input and checking phase (including line sectioning)	-	0.8 minutes
Execution phase	-	3.0 minutes
Output phase (printer 'graphical' output)	-	5.0 minutes
		12.3 minutes

The system has the facility to carry out a number of runs on different input data one after the other without the necessity for the pre-execution phase to be run before each one. It must, of course, be executed prior to the first run in any group. If the simple numerical output option had been selected, the output phase would only have required some 30 seconds.

The actual computing time (data input phase and pre-execution phase) of 3.8 minutes would probably transform to under one minute if a faster machine - e.g. a machine of the size of, say, I.B.M. 360/40 or I.C.L. 1903 - were to be used.

6.3 Example of laboratory simulation

Virtually the only way of validating a program of this type is by direct comparison between computed and experimental results. Some years ago this would have proved to be very difficult in view of the small transit times and extremely fast risetimes required from the measuring equipment. The advent of the sampling oscilloscope and of the still more sophisticated time domain reflectometer now make transient measurements on practicably sized laboratory models relatively straightforward. Published work on the time domain reflectometer gives additional information on this type of equipment [41].

A number of laboratory models were constructed and these consisted for the most part of lengths of transmission line whose characteristics could be determined with reasonable accuracy. Additional components were introduced as necessary to enable most of the program functions to be tested. Line lengths were at most only about ten metres with corresponding transit times of around 30 nanoseconds. One major problem at these very high frequencies is the difficulty of making, say, a purely resistive termination to a line. Lead inductances and stray capacitances can introduce considerable distortions in the expected transients.

In all cases good agreement was reached between the experimental and computed waveforms. An example of one of these tests - designed to validate the routines involved in handling coupled coils - is shown in fig. 6.8. The lower portion of this figure illustrates the network which was used, simply two lengths of transmission line (co-axial cable) together with a purpose-made air cored coil set. The upper part of the figure shows the very

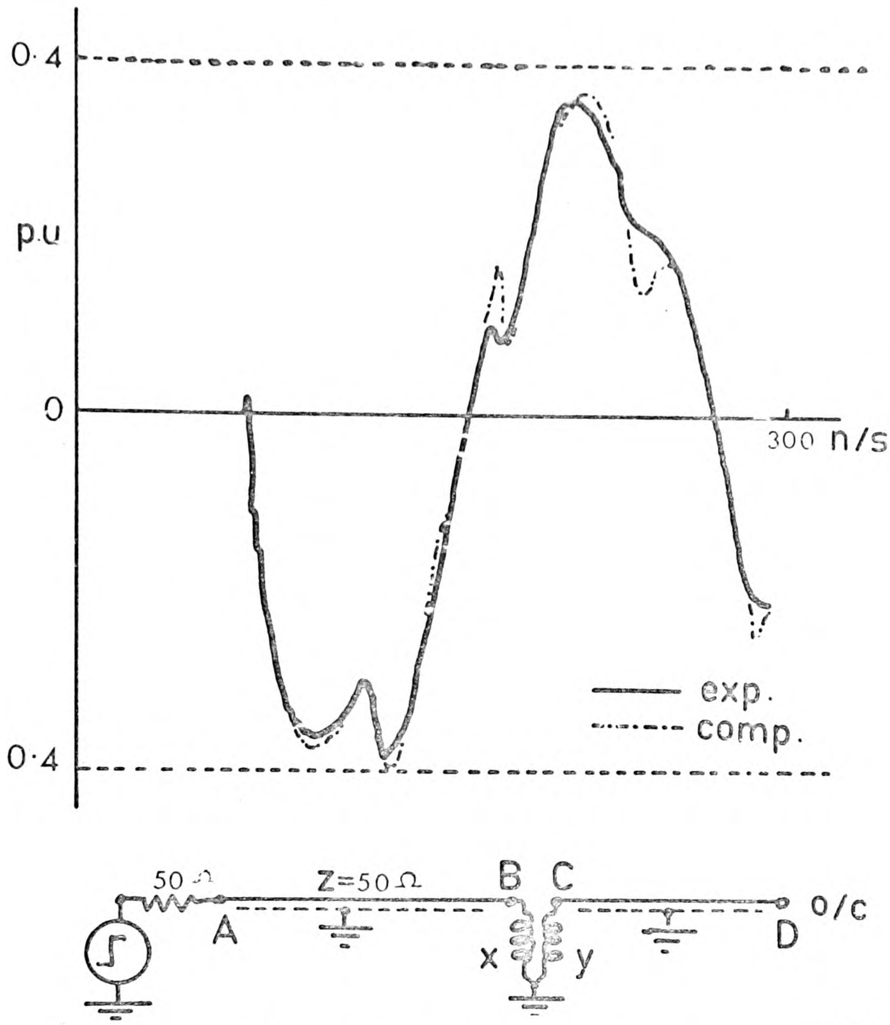


FIG. 6.8

Line AB - T = 50 nanoseconds
R = 2.3 ohms

Line CD - T = 25 nanoseconds
R = 1.2 ohms

Coils - Section 'x' L = 1.43 H
k = 0.52 'y' L = 1.52 H

good correlation which was obtained. The approximation inherent in the calculation procedure for the coupled coils has not significantly distorted the results.

In order to investigate the performance of the program in cases where mutual coupling exists between line conductors, an air-insulated model multi-conductor line was constructed. Work with this line will be described in the following chapter.

6.4 Capacitively loaded transmission lines - an approximation

In the design of high speed pulse and logic circuitry, it is often necessary to take account of the finite propagation delays produced by even short lengths of transmission line connecting network elements. The terminations of such lines are predominantly resistive, but there is often a pronounced capacitive component - due to the input capacitances of gates, etc. - which can have a marked effect on the transient response of the system as a whole.

Work on the computation of transients in networks of this type by Wassel has already been mentioned [20], and it will be recalled that a Laplace transform approach was used in his case. These calculations were apparently carried out under the sponsorship of a leading computer manufacturer and may thus be regarded as being of practical importance.

The SUSAN system has been used for investigating lines having this form of R-C termination and as a result the work of Wassel has been confirmed. At the same time the investigation has been continued to the point where it has proved possible to formulate an approximation which considerably assists in the approximate calculation of the transient responses of this type of network. This approximation has been described by the writer in the literature [42].

The basic network configuration investigated is that shown in fig. 6.9. Transient calculations in such a system are considerably more complex than in the corresponding case where the terminating capacitors may be ignored. Indeed in this latter case the calculations are so straightforward that the basic graphical method may be employed directly without the use of a computer.

The basis of the approximation - detailed more fully in subsequent paragraphs - is that a system such as that shown in fig. 6.9 may be replaced by a similar network having purely resistive terminations, but whose transmission line is longer than the original line.

Fig. 6.10 shows a typical response, as computed using SUSAN for the network. This illustrates the form of the receiving end voltage (node 2) following a step function applied at node 1. The response has distinct steps preceeded by a pre-step oscillation. The number of cycles of pre-step oscillation is related to the number of times the disturbance has been reflected from the receiving end termination.

Some forty runs were carried out on networks of this form for various combinations of R and C loading and the following conclusions were reached :-

6.4.1 For a single line (e.g. fig.6.9) where there is a combination of resistive and capacitive loading, the amplitude of the 'steps' in the response to a step function input are determined by the resistances present and are independent of capacitance.

6.4.2. The effect of the lumped capacitors is to artificially 'lengthen' the transit time of the line so that steps occur later than for purely resistive loading. Also a degree of pre-step oscillation occurs, the number of cycles of which equals the number of the reflection.

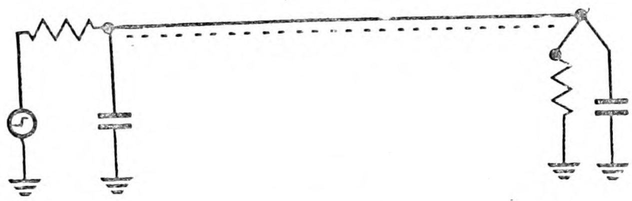


FIG.6.9

SUSAN I - OUTPUT

MESSAGE (CPU #S30) - RUN TERMINATED MANUALLY

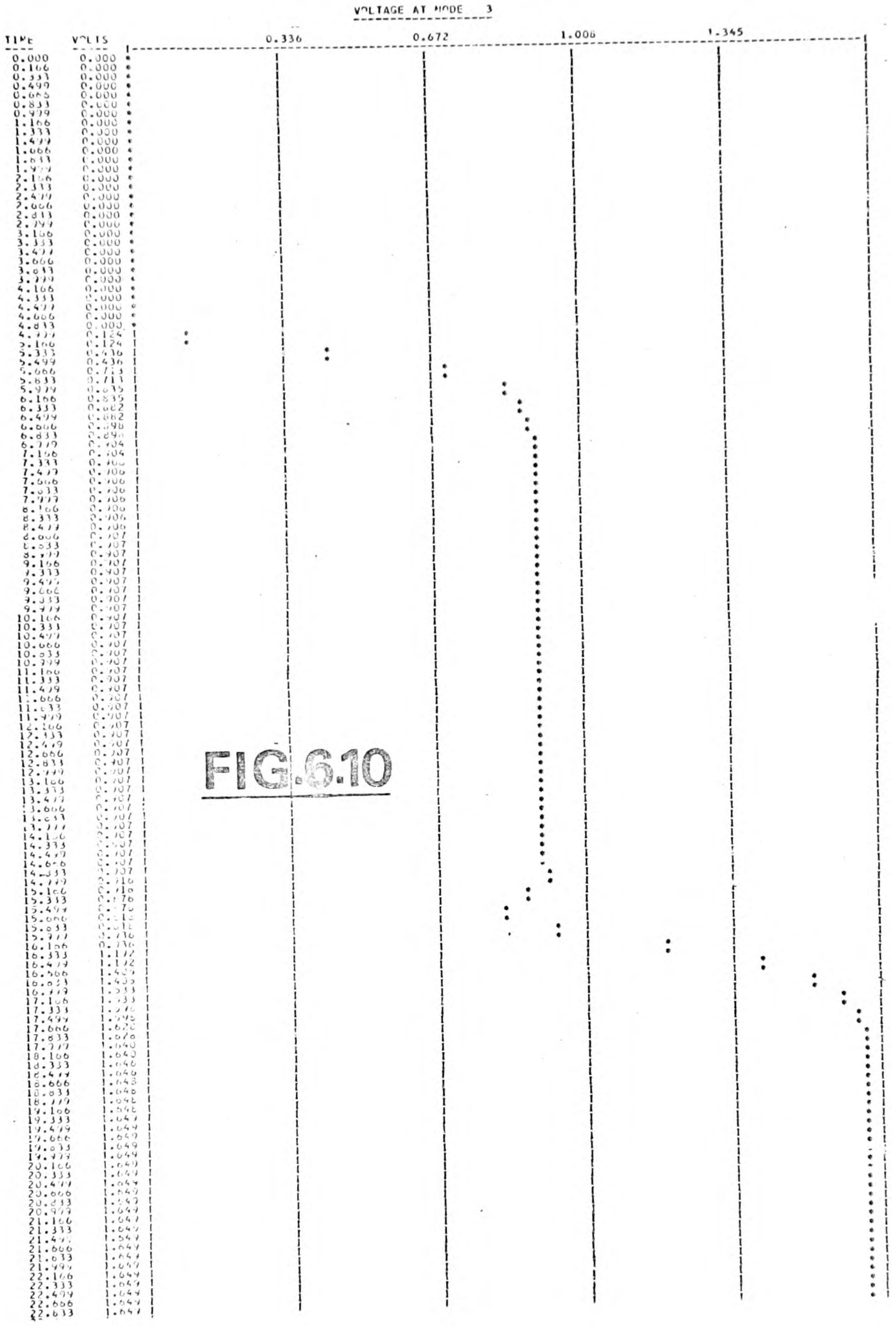


FIG.6.10

6.4.3 The degree of symmetry of the capacitive loading has some effect on the lengthening of the line. A ratio of sending end to receiving end time constants of unity produces the most pronounced effect, but the difference in the effects produced by extremes of symmetry in loading was only about 12%. The minimum effect is found to occur when all the capacitance is concentrated at one end. Fig. 6.11 shows the variation of the lengthening effect (defined as 'capacitive hold time') against ratios of T1 and T2 where these are the time constants of the sending and receiving ends respectively. 'Capacitive hold time' is defined as follows :-

$$\text{capacitive hold time (c.h.t)} = (t - (2m + 1).T')/Tav \quad \ldots\ldots\ 6.1$$

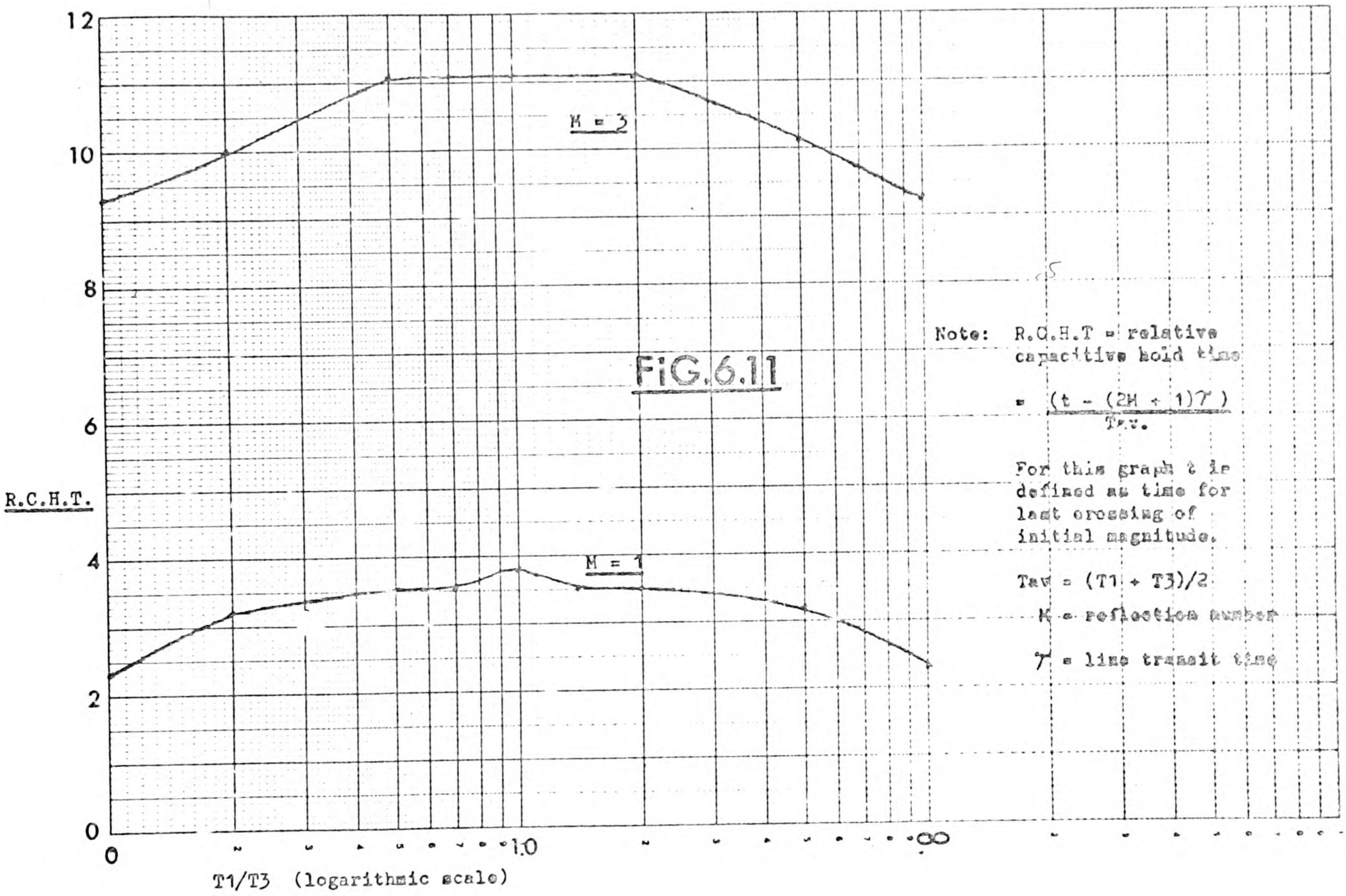
here m is the number of times the pulse has been reflected
T' is the line transit time
Tav is the average time constant of the sending and receiving ends of the line.

The time 't' may be defined in a number of ways, and is the point in time when each step in the response is deemed to occur. The writer's definition of t is the time when each step has reached 65% of its final amplitude, measured from a baseline of the amplitude of the preceeding step.

In the computation of the time constant at a nodal point in the network, it is necessary to take the surge impedances of all transmission lines terminating at that point into consideration as well as any lumped resistance which may be present. In the general case of n lines having surge impedances Z1, Z2 etc. terminating at a node together with a resistance R and capacitance C, the time constant of that node is given by :-

$$T = (1/R + 1/Z1 + 1/Z2 \quad \ldots \quad +1/Zn)^{-1}.C \quad \ldots\ldots\ 6.2$$

With the writer's definition of the 't' parameter in equation 6.1, it is found that a graph of c.h.t. against the reflection number, m, is a straight line. Furthermore this line has the simple equation:-



12
10
8
6
4
2
0
R.C.H.T.
M = 3
M = 1
FIG.6.11
0
1.0
00
T1/T3 (logarithmic scale)
Note: R.C.H.T = relative capacitive hold time
$= \frac{(t - (2M + 1)\tau)}{T_{av.}}$
For this graph t is defined as time for last crossing of initial magnitude.
Tav = (T1 + T3)/2
M = reflection number
τ = line transit time

$$c.h.t. = 4m + 2$$
$$= 2.(2m + 1) \quad \ldots\ldots\ldots \quad 6.3$$

Combining equations 6.1 and 6.3 we have that :-

$$2.(2m + 1).Tav = t - (2m + 1).T' \quad \ldots\ldots \quad 6.4$$

or $$t = (2m + 1).(T' + 2.Tav) \quad \ldots\ldots\ldots\ldots\ldots \quad 6.5$$

Now suppose that the network has no capacitive elements and the transit time of the transmission line is T". In that case steps in the response would be expected to occur at times t where :-

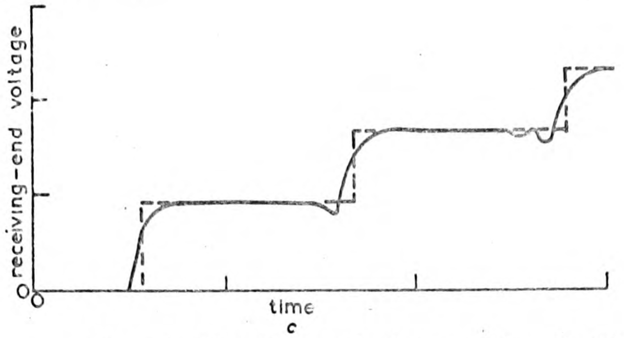
$$t = (2m + 1)T" \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 6.6$$

By comparison of equations 6.5 and 6.6, it may be inferred that if the transit time in the capacitively loaded network is increased from T' to T" (i.e. from T' to T +2.Tav) and the capacitors are removed then the response of the simplified network approximates closely to the true response of the original system. Fig. 6.12 shows the actual network response similar to fig. 6.10, but with the approximate response superimposed. It is found that the steps will always occur at times corresponding to approximately 65% of each step amplitude.

6.4.4 Application to more complex systems

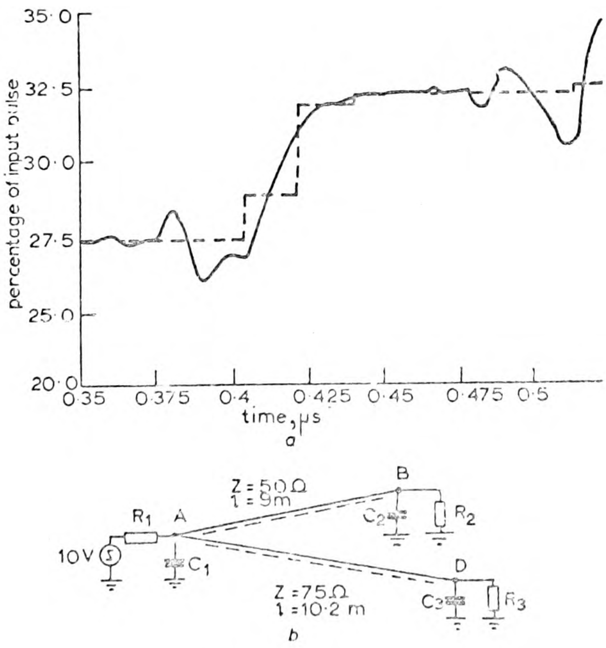
From the networks considered it was concluded that the approximation outlined above is valid independently of transmission line transit time, or of the magnitudes of the termination time constants. However when the ratio T'/Tav falls below about 3, there is a tendancy for pre-step oscillation to merge with the previous step after one or two reflections. The response then rapidly deteriorates to a simple exponential. In such cases the presence of the line may be ignored.

The approximation is valid for networks containing more than one transmission line, and which have no degree of symmetry, provided



Network with resistive terminations Fig. 6.12

——— true response
— — — approximate response



Unsymmetrical network

——— true response
— — — approximate voltage at node B

$C_1 = 60$pF $R_1 = 500\Omega$
$C_2 = 75$pF $R_2 = 600\Omega$
$C_3 = 90$pF $R_3 = 1400\Omega$
Line propagation velocity = 60% of that in Fig. 1c

Fig. 6.13

that they are not so complex as to preclude the formation of distinct steps in their time responses to step functions. Such an unsymmetrical network is shown in fig. 6.13 which also shows a portion of the true response at node B to a step function applied at node A at time 0.

The time constants T1, T2 and T3 for nodes A, B and D are 1.695, 3.46 and 6.4 nanoseconds respectively. To apply the approximation to this network, it is thus necessary to increase the transit times of lines AB and AD by (T1 + T2) and (T1 + T3) respectively. Hence the transit time of AB is increased by 5.155 nanoseconds and that of line AD by 7.095 nanoseconds. If this increase is regarded as one of length, it means that the length of line AB must be increased from 9.0 to 9.93 metres, and that of line AD from 10.2 to 11.475 metres. The capacitors are removed and the resistive elements remain unchanged. Fig. 6.13 shows, superimposed on the true response, the response of the modified network and illustrates the general agreement between the amplitudes of the steps and the times at which they occur.

6.5 Transmission Tower 'Footing' Resistance

The final example considered here concerns an investigation into the effect of tower footing resistance on the voltage stress across the conductor insulation due to a direct lightning strike on a transmission line tower. This example serves to illustrate one application of SUSAN in the power engineering field.

In studies of this kind the shape of the voltage/time transient curve is of importance as well as the magnitude of the peak voltage appearing across the insulation. Flashover may well occur if this curve intersects the voltage/time breakdown characteristic of the insulation, even though this intersection takes place at a point beyond the peak value of voltage [43].

Consider the transmission tower shown in fig. 6.14. The applied lightning stroke has a waveform with a relatively small risetime, in the order of 1.0 microseconds. Therefore the time of interest in the problem will be such that reflections from other towers need not be considered, hence the earth wires and line conductors are replaced by resistors to ground. The tower is thus reduced to the model shown in fig. 6.14b, and then by symmetry to fig. 6.14c. The following nomenclature has been used :

Z1 - surge impedance of the stroke

Z2 - surge impedance of the top section of the tower

Z3 - surge impedance of the cross-arms

Z4 - surge impedance of the tower body

R1 - (earth wire surge impedance)/2

R2 - (line conductor surge impedance)/2

C - insulator string self capacitance

Rg - tower footing resistance

T1 - transit time along one cross-arm

T2 - transit time along tower body

Z'3 - Z3/2

C' - 2.C

The accepted form of a lightning stroke is that of a double exponential which is available directly in the SUSAN system. Alternatively a double ramp function would provide an acceptable forcing function. For these calculations a standard 1/50 impulse waveform was employed.

In order to be able to use the 'standard' node types available in the program, the final system in fig. 6.14c has to be modified slightly to that shown in fig. 6.15. Two additional sections of transmission line have been added though this new network is electrically identical with that which it replaces.

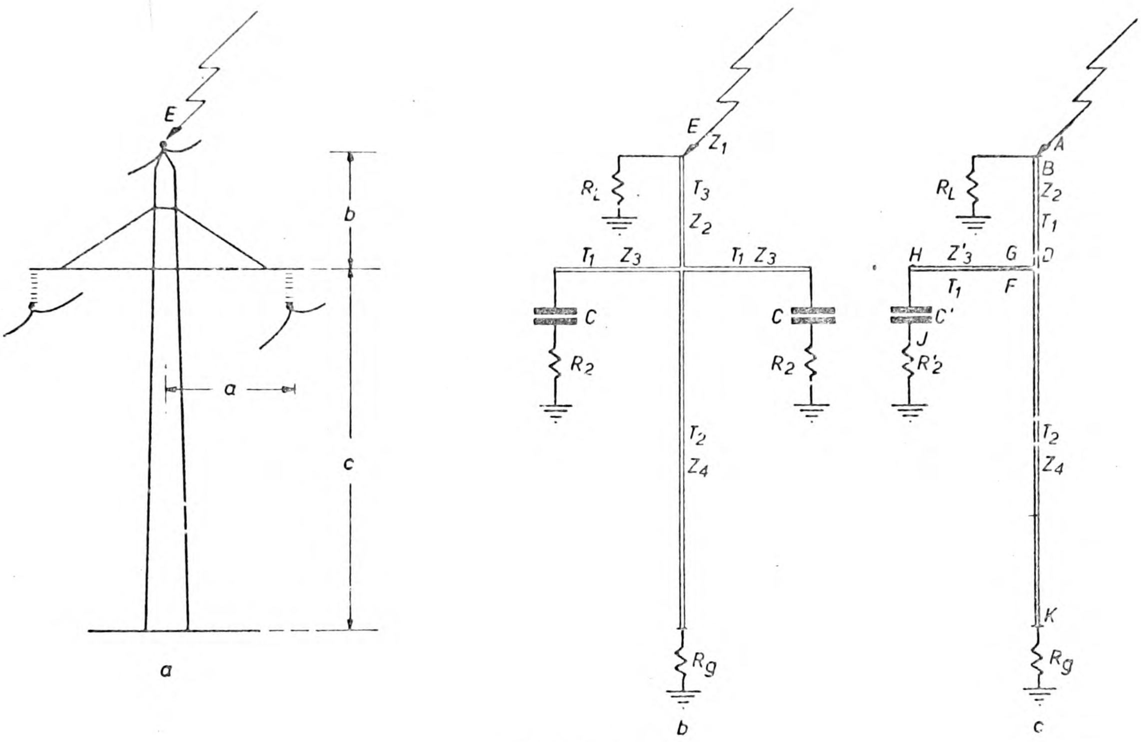

E
b
a
c
a
E
Z1
RL
T3
Z2
T1 Z3
T1 Z3
C
R2
C
R2
T2
Z4
Rg
b
A
B
RL
Z2
T1
H
Z'3
G
D
T1
F
C'
J
R'2
T2
Z4
K
Rg
c

FIG.6.14

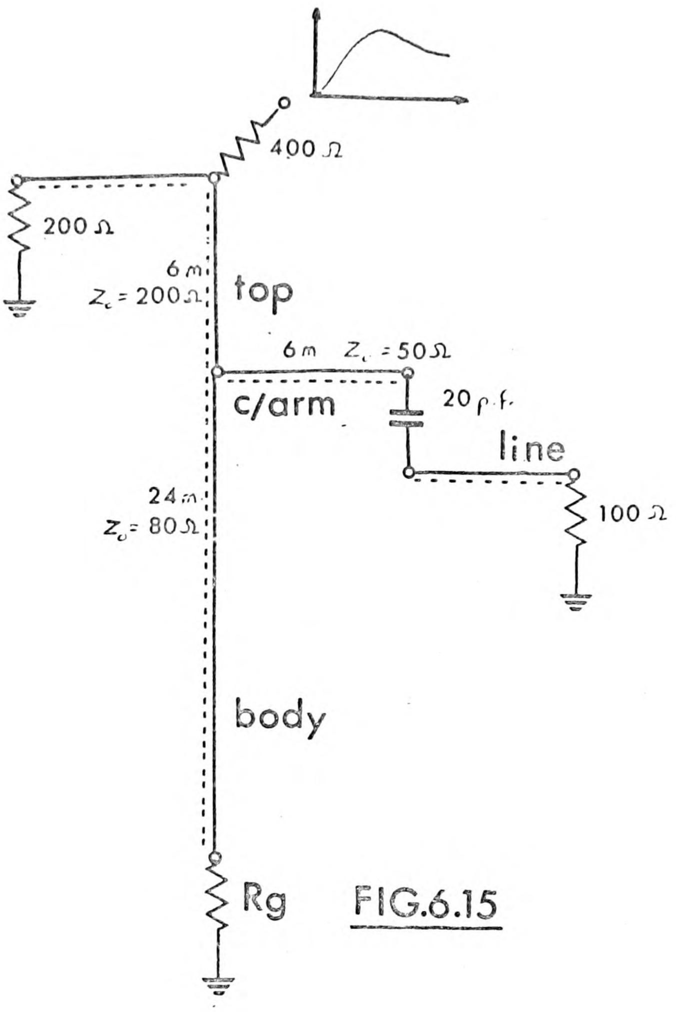

400 Ω
200 Ω
6 m
$Z_c = 200\,\Omega$
top
6 m $Z_c = 50\,\Omega$
20 p.f.
c/arm
line
100 Ω
24 m
$Z_0 = 80\,\Omega$
body
Rg

FIG.6.15

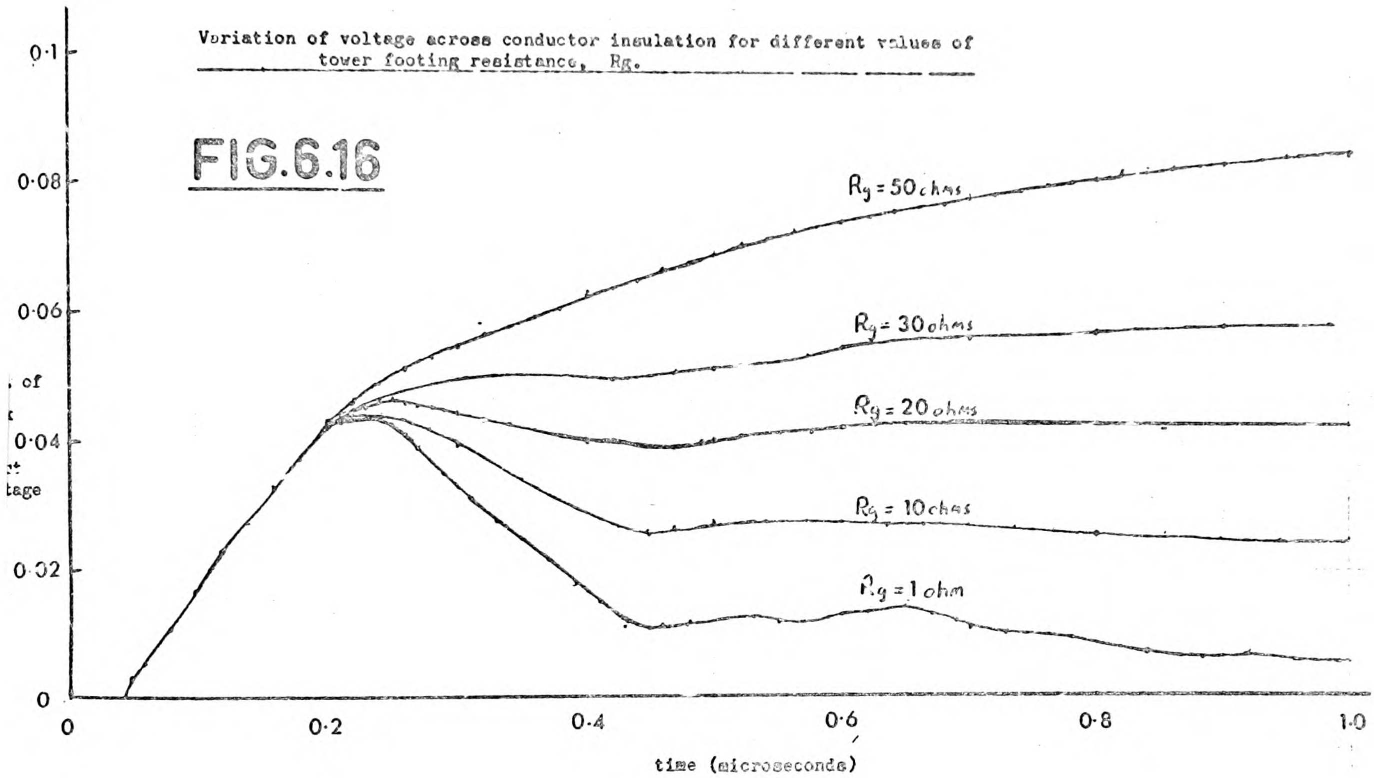
Fig. 6.16 indicates the type of results obtained for various values of the footing resistance Rg. Results such as these assist in the specification of a minimum acceptable footing resistance and are of practical importance when lines are strung in regions where good earthing of towers is difficult to achieve.

A system similar to the one described here was investigated by Arlett[37] except that in his case the forcing function was a double ramp rising to a peak value in 0.2 microseconds, then falling again to 0.5 of the peak in 20 microseconds.

Whilst the general form of the results as shown in fig. 6.16, and those proposed by Arlett are similar, Arlett reports a peak voltage across the insulator of some 23% of the peak input level (for Rg = 50 ohms). Fig. 6.16 shows that for the same value of Rg, the maximum voltage attained in the present case is some 9% of the peak level.

This discrepancy is thought to be due to Arlett's figures being incorrect, a fact supported by a simple examination of the network (fig. 6.15). It is seen that under 'steady state' conditions, with a step function or 'slow' double exponential input, the circuit degenerates to the source impedance (400 ohms) in series with a parallel combination of 200 ohms and Rg (=50 ohms). Under these conditions the voltage across the line insulator capacitance approximates to that across Rg which is about 0.09 p.u. of the input value. Since this is a hypothetical example, no field test results are available for comparison.

Other examples of the use of programs such as this in the power engineering field would include the computation of circuit breaker restriking transients, investigation of circuit breaker operation and positioning and analyses of the effects of the inclusion of cable sections into predominantly air-insulated transmission systems.



Variation of voltage across conductor insulation for different values of tower footing resistance, Rg.
FIG.6.16
Rg = 50 ohms
Rg = 30 ohms
Rg = 20 ohms
Rg = 10 ohms
Rg = 1 ohm
0·1
0·08
0·06
0·04
0·02
0
0·2
0·4
0·6
0·8
1·0
time (microseconds)

7. MUTUAL LINE COUPLING

7.1 Introduction

During the past few years much work has been directed towards this problem of transient calculations in systems which have mutually coupled transmission lines. There has been a natural progression from the production of programs suitable for single phase networks only to those designed for polyphase work. Not surprisingly most interest has centred in the power systems field and little work has been done with electronic networks in mind.

Transient programs which have been developed so far, and there have really been very few of these, have tended to be either 'single phase' or 'multiphase' in their application. The fact that SUSAN has the capability to handle networks containing both uncoupled and coupled lines within the same problem makes it virtually unique at the time of writing.

The background to the mutual coupling problem was first considered in detail by Bewley [44] as early as 1933. He introduced the concept of 'mutual surge impedance' which is important in the approach finally used in SUSAN. More recently other workers have advocated the use of 'symmetrical component' procedures to overcome the problems inherent in dealing with mutual coupling, and several of the programs written for polyphase work in the past few years have used this approach. Symmetrical components are not, of course, new or unfamiliar and as early as 1932 Fallou[45] suggested that they might be applied to transient problems. These ideas were reviewed in 1963 by Weedepohl[46] and similar techniques were employed by Thoren and Carlsson[47], by Dommel[38] and by Arismunandar and Price[48] in their polyphase programs.

7.2 Mutual coupling and the graphical method

Both the 'mutual surge impedance' and the 'symmetrical component' procedures were investigated with a view to their inclusion in SUSAN. The technique finally adopted is based on the former idea, though, as will be demonstrated now, the latter process may also be used. The following analysis forms the basis of a paper by the writer published in the literature[49].

This analysis considers that networks are composed of groups of mutually coupled lines, and that there are N conductors in each line group. Whilst all the conductors forming a single line group are coupled together, there is no mutual coupling between one line group and another. If N = 1, then the problem degenerates to the simple single conductor case which has been considered previously.

This case is illustrated in fig. 7.1 where there are m line groups (each of one conductor) terminating at node k. The equation for the voltage at node k at time 1 in terms of the voltages at each of the surrounding nodes one time unit in the past has already been obtained as :-

$$V_{k,1} = (A_0.Y_{ak} + B_0.Y_{bk} + \dots + M_0 Y_{mk} + e(t)/R)/(Y_{kk} + 1/R) \quad \dots\dots 7.1$$

$$\text{where } A_0 = (V_{a,0} - Z_{ak}.i_{ak,0}) \text{ etc.} \quad \dots\dots\dots\dots 7.2$$

Here the termination at node k is assumed to be a simple voltage source.

Now consider the N conductor case as shown in fig. 7.2. The line groups ak, bk, mk etc. have mutual coupling between conductors. Considering a single line group mk, and assuming that the conductor series resistances and shunt conductances are negligable, the general wave equation representing transient voltages on the line in the time domain can be written as:-

$$d^2V/dx^2 = (L_{mk})(C_{mk})d^2V/dt^2 \quad \dots\dots\dots\dots 7.3$$

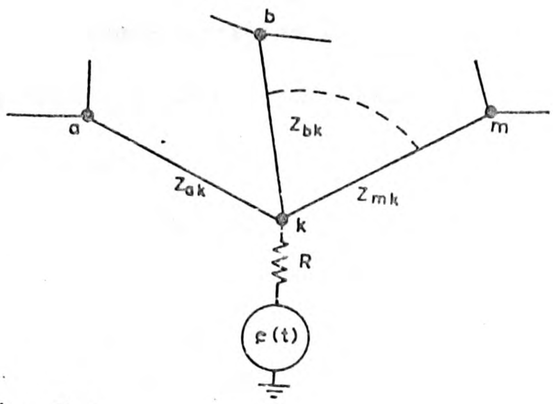

b
a
m
Zbk
Zak
Zmk
k
R
e(t)

Fig. 7.1

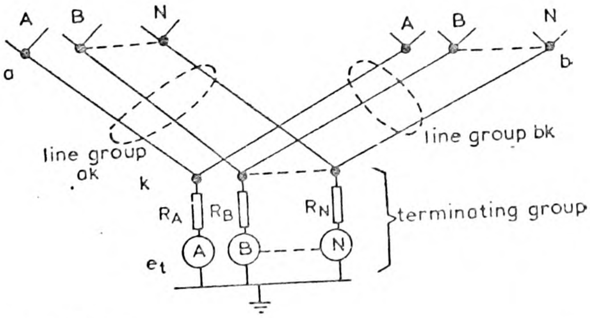

A
B
N
A
B
N
a
b
line group
ak
line group bk
k
RA
RB
RN
terminating group
A
B
N
et

Fig. 7.2

In this equation (L_{mk}) and (C_{mk}) are the inductance and capacitance coefficient matrices respectively. The solution of equation 7.3 is complicated by the fact that the product $(L_{mk})(C_{mk})$ will contain non-zero off-diagonal terms.

Transformation of the complete system into a modal domain by the use of a suitable transform operator results in this $(L_{mk})(C_{mk})$ product becoming a diagonal matrix. This implies that the conductors within the line group will now appear to be mutually independent.

The transformation matrix, (T), can be selected on the basis of the degree of unbalance of this line group.

If the voltages in the untransformed system are contained in the vector (V), and the transformed voltages are given by (V'), we have that :-

$$(V) = (T)(V') \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ 7.4$$

Hence :-

$$(T).d^2V'/dx^2 - (L_{mk}).(C_{mk}).(T).d^2V'/dt^2 = 0 \qquad \ldots\ldots\ldots\ 7.5$$

Multiplying by the inverse of the transforming matrix gives :-

$$d^2V'/dx^2 - (T)^{-1}.(L_{mk}).(C_{mk}).(T).d^2V'/dt^2 = 0 \qquad \ldots\ldots\ 7.6$$

For these equations to be independent, the product :-

$(T)^{-1}.(L_{mk}).(C_{mk}).(T)$ must be a diagonal matrix - i.e. have zero off-diagonal terms.

$$\text{Let } (A) = (L_{mk}).(C_{mk}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 7.7$$

and let the diagonal matrix be (K) where
$$(K) = \begin{pmatrix} k1 & 0 & 0 \\ 0 & k2 & 0 \\ 0 & 0 & k3 \end{pmatrix} \qquad \ldots\ldots\ldots\ 7.8$$

then :-
$$(T)^{-1}.(A).(T) = (K) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 7.9$$

from which we have :-

$$(A).(T) - (T).(K) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 7.10$$

Expanding the first row of equation 7.10 gives :-

$$A11.T11 + A12.T21 + \ldots\ldots + A1n.Tn1 = T11.k1 \quad \ldots . \; 7.11$$

Thus:-

$$(A11 - k1).T11 + A12.T21 + \ldots + A1n.Tn1 = 0 \; . \; 7.12$$

If T11, T12 etc. are non zero, and for a non-trivial solution then:-

$$\det.((A)-(K)) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 7.13$$

It should be noted that equation 7.13 is independent of (T). For this equation to be valid, it therefore follows that the non-zero diagonal elements of (K) must be the eigenvalues of (A).

The columns of the transformation matrix (T) are the eigenvectors of (A) and one element in each column may be arbitrarily specified. It therefore follows that for any given degree of unbalance - represented by a given set of eigenvalues of the (A) matrix - there are an infinite number of transformation matrices which can be employed.

The 'conventional' symmetrical component transformations due initially to Fortescue and latterly to Clarke are specific examples of this eigenvalue problem applied to particular cases - cases where the conductors are arranged in a symmetrical pattern. Indeed use of the Clarke transformation - which has the advantage of not containing complex terms - may be readily made in conjunction with the basic graphical method for the solution of simple multi-phase transient problems. A paper by Arlett and the writer[50] illustrated this operation.

In the single conductor case the line surge impedance is defined as $(L_{mk}/C_{mk})^{\frac{1}{2}}$. By analogy when N is greater than 1, we may define a surge impedance matrix Z_{mk} where :-

$$(Z_{mk}) = ((L_{mk}).(C_{mk})^{-1})^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 7.14$$

This too may be diagonalised via the use of the transform matrix (T) so that:-

$$(Z'_{mk}) = (T)^{-1}(Z_{mk}).(T) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 7.15$$

We thus have that :-

$$Z'_{mk} = \begin{pmatrix} Z'_{mkA} & 0 & \ldots\ldots\ldots & 0 \\ 0 & Z'_{mkB} & \ldots\ldots\ldots & 0 \\ \ldots & & \ldots\ldots\ldots & . \\ 0 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & Z_{mkN} \end{pmatrix} \quad \ldots\ldots\ldots\ldots \; 7.16$$

A surge admittance matrix may also be defined as (Y_{mk}) where :-

$$(Y_{mk}) = (Z_{mk})^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 7.17$$

In fig. 7.2 the transmission lines are terminated by a 'terminating group' which, for the configuration shown, has an admittance matrix of the form :-

$$Y_{term} = \begin{pmatrix} 1/R_A & 0 & \ldots & 0 \\ 0 & 1/R_B & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & 1/R_N \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 7.18$$

In general Y_{term} will have non-zero off diagonal terms and might not be sparse as in this case. It will, of course, normally be symmetric. The transformed matrix, Y'_{term} will neither be symmetrical nor sparse :-

$$(Y'_{term}) = (T)^{-1}.(Y_{term}).(T) \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 7.19$$

With all voltages and currents in modal form, we have by analogy to equation 7.1 :-

$$(V'_{k,1}) = \frac{((Y'ak).(A'O) + (Y'bk).(B'O) + \ldots + (Y'mk).(M'O) + (Y'term).(e't))}{(Y'ak) + (Y'bk) + \ldots\ldots + (Y'mk) + (Y'term)}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \; 7.20$$

In this equation :-

$$(A'0) = (V'a0) - (Z'ak).(i'ak,o) \quad \text{etc.} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 7.21$$

Each term in these equations is, itself a matrix and, for example, in accordance with the previous notation $i'_{ak,ON}$ is the current at time 0 from node a to node k along modal line N of the line group ak. The forcing function, e't is derived from the forcing function in the real domain by a standard application of the transform matrix. Thus :-

$$(e't) = (T).(e_t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 7.22$$

The computation is carried out entirely in the modal domain, conversion of voltages and currents to the real domain only being carried out when results are required.

The relations described above show that this method of approach should be theoretically useful. In practice, however, its was not employed for a number of reasons. These may be summarised as follows :-

7.2.1

The transformation matrix, (T), is determined by computation of the eigenvectors of the $(L_{mk}).(C_{mk})$ matrix product. This calculation, whilst possible, produced difficulties due to the very small magnitudes of the numbers involved. As an alternative to actually computing (T) on the basis of the physical dimensions of the conductors forming the line group, some workers, especially Dommel[36], have suggested that each line group be considered as being symmetric - that is all the mutual inductance coefficients are equal, as are all the self inductances and so on. If this is done then any one of the 'standard' symmetrical component transforms can be used, or, even more simply, we may have that :-

$$(T) = \begin{pmatrix} 1 & 0 & .. & 0 \\ 0 & 1-N & .. & 0 \\ .. & .. & & .. \\ 0 & ... & & 1-N \end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 7.23$$

Though this assumption simplifies the analysis, it may be really justified only in the case of cables and similar structures where the earth plane surrounds the line conductors. In addition the advantage of being able to make an allowance for the earth plane being a non-perfect conductor - by assuming that the distance from a conductor to its voltage image is different from that to its current image - is lost.

7.2.2 Since the transform matrix (T) is 'tailor made' to suit the geometry of the line groups, this implies that if the same transform is to be used throughout the network, then each line group must be geometrically identical. The only alternative is to compute a separate transform matrix for each type of line group and then establish 'interfaces' at each node so that parameters may be transformed from one modal domain to another as required. This technique has been investigated by Arismunander et. al.[48] who succeeded in computing transients in a multi-phase network containing line sections requiring the use of several transforms. Even here, however, discrepancies of up to 25% occurred between computed results and field tests.

7.2.3 The use of a transform technique can produce difficulties in cases where uncoupled lines are included in the same network as lines with mutual coupling. In the same way terminating groups which initially have no coupling or mutual terms become coupled when operated on by (T).

7.2.4 The amount of computation, and consequently the processor time and storage required, are considerably more using this transform idea than is the case with alternative techniques.

7.3 Mutual Coupling in the SUSAN system

The transform technique outlined above removes the impedance coupling between one line and the next by diagonalising the surge impedance matrix, (Z_{mk}) .

The velocities of propagation on the modal lines will, of course, differ from those in the real system. On the modal network there will be N different velocities - one on each line - whilst in the real network there are N^2 velocities since each conductor is constrained to propagate N waves, each having a different speed.

If, in the real network, all waves travelled at the same velocity, then the problem of treating mutual coupling is much simplified. Given that a disturbance, started at one end of a line group, will travel down all the conductors with the same velocity, mutual coupling between these conductors can be described simply in terms of mutual impedances between each line.

Bewley has shown that multivelocity waves cease to exist - and therefore that waves on all conductors travel with the same velocity - if two assumptions are made. These are:-

(a) The conductors have no internal magnetic field

and (b) The earth plane is 'perfect' - i.e. the image axis for current is coincident with that for voltage.

These two assumptions can reasonably be made for a whole range of practical problems. In light current situations the earth plane is frequently a very good conductor, whilst the very high frequencies encountered in the transient response imply that 'skin effects' in the line conductors will be very pronounced and consequently that the currents actually flowing deep within the line material will be quite small. The situation is not quite so favourable in the power systems field, however, since the frequencies dominant in the responses are not so high, and more important, earths are no so 'perfect' as in the former case.

The enormous advantages of this approach are that it allows mutually coupled line sections to the incorporated very easily into networks of uncoupled lines, that the amount of computation required is less than with the former process, and that the technique 'blends' more easily with the logic of the existing SUSAN system.

In the present version of SUSAN coupling is restricted to two lines only. This was done purely on account of the availability of storage in the 1130 computer and there is theoretically no reason why the approach should not be extended to systems of N phases.

There is, in reality, a direct analogy between single phase systems and polyphase systems - if all waves are assumed to travel at the same velocity - wherein single terms in the equations for single phase systems are replaced by matrices in the polyphase case.

Hence for a set of coupled lines we have :-

$$(V) + (Z).(I) = (\text{const.}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 7.24$$

$$\text{and} \quad (V) - (Z).(I) = (\text{const}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 7.25$$

$$\text{where} \quad Z = \begin{pmatrix} Z11 & Z12 \\ Z21 & Z22 \end{pmatrix} \quad \text{for a 2 phase system} \quad . \quad 7.26$$

Consider a set of two coupled lines terminating at nodes a and k as shown in fig. 7.3. For this line group we have :-

$$(V)_{k,1} + (I)_{ka,1}.(Z_{ak}) = (V)_{a,0} - (I)_{ak,0}.(Z_{ak}) \quad \ldots.. \quad 7.27$$

expanded this becomes :-

$$\begin{pmatrix} V1 \\ V2 \end{pmatrix}_{k,1} + \begin{pmatrix} Z11 & Z12 \\ Z21 & Z22 \end{pmatrix} \begin{pmatrix} i1 \\ i2 \end{pmatrix}_{ka,1} = \begin{pmatrix} V1 \\ V2 \end{pmatrix}_{a,0} - \begin{pmatrix} Z11 & Z12 \\ Z21 & Z22 \end{pmatrix} \begin{pmatrix} i1 \\ i2 \end{pmatrix}_{ak,0} \quad \ldots.. \quad 7.28$$

This last equation provides the basis for the practical formulation actually used in the program.

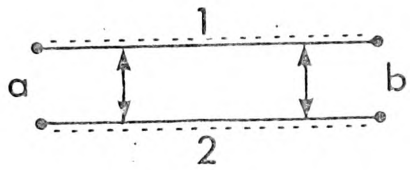


FIG.7.3

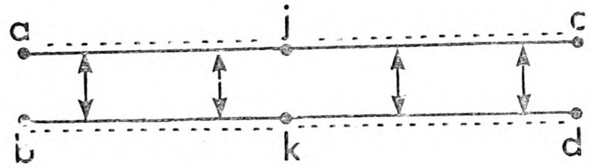


FIG.7.4

Considering the slightly extended network shown in fig. 7.4, letting $Z11 = Z22 = Z$, and similarly letting $Z12 = Z21 = Zm$, and additionally remembering that the algebraic sum of currents into nodes j and k at any one time is zero, we have :-

$$\begin{pmatrix} Z & 0 & Z & 0 & 1 & 0 \\ 0 & Z & 0 & Zm & 1 & 0 \\ Zm & 0 & Z & 0 & 0 & 1 \\ 0 & Zm & 0 & Z & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} i_{ja,1} \\ i_{jc,1} \\ i_{kb,1} \\ i_{kd,1} \\ V_{j,1} \\ V_{k,1} \end{pmatrix} = \begin{pmatrix} Va,o - Z.i_{aj,o} - Zm.i_{bk,o} \\ Vc,o - Z.i_{cj,o} - Zm.i_{dk,o} \\ Vb,o - Z.i_{bk,o} - Zm.i_{aj,o} \\ Vd,o - Z.i_{dk,o} - Zm.i_{cj,o} \\ 0 \\ 0 \end{pmatrix}$$

.... 7.29

The left hand side coefficient matrix may be pre-computed and subsequently inverted. In the event that the line sections are not mutually coupled - e.g. if jc and kd had been coupled, but aj and bk were not - then the coefficient matrix and the right hand side requires modification by elimination of the relevant mutual terms.

The main problem which developed when this analysis was incorporated into the basic SUSAN system, was one of being able to readily identify currents and voltages on the 'other' line whilst continuing to adopt the policy that networks must be easy to code from the user's point of view. From a programming standpoint the logic required to perform the calculations expressed in equation 7.29, within the existing framework of the program proved to be tortuous but nevertheless possible.

7.4 Calculations in a coupled system

In an attempt to validate the operation of the program in cases where mutual coupling exists, direct comparison was made between computed and laboratory results. The laboratory system consisted of a model transmission

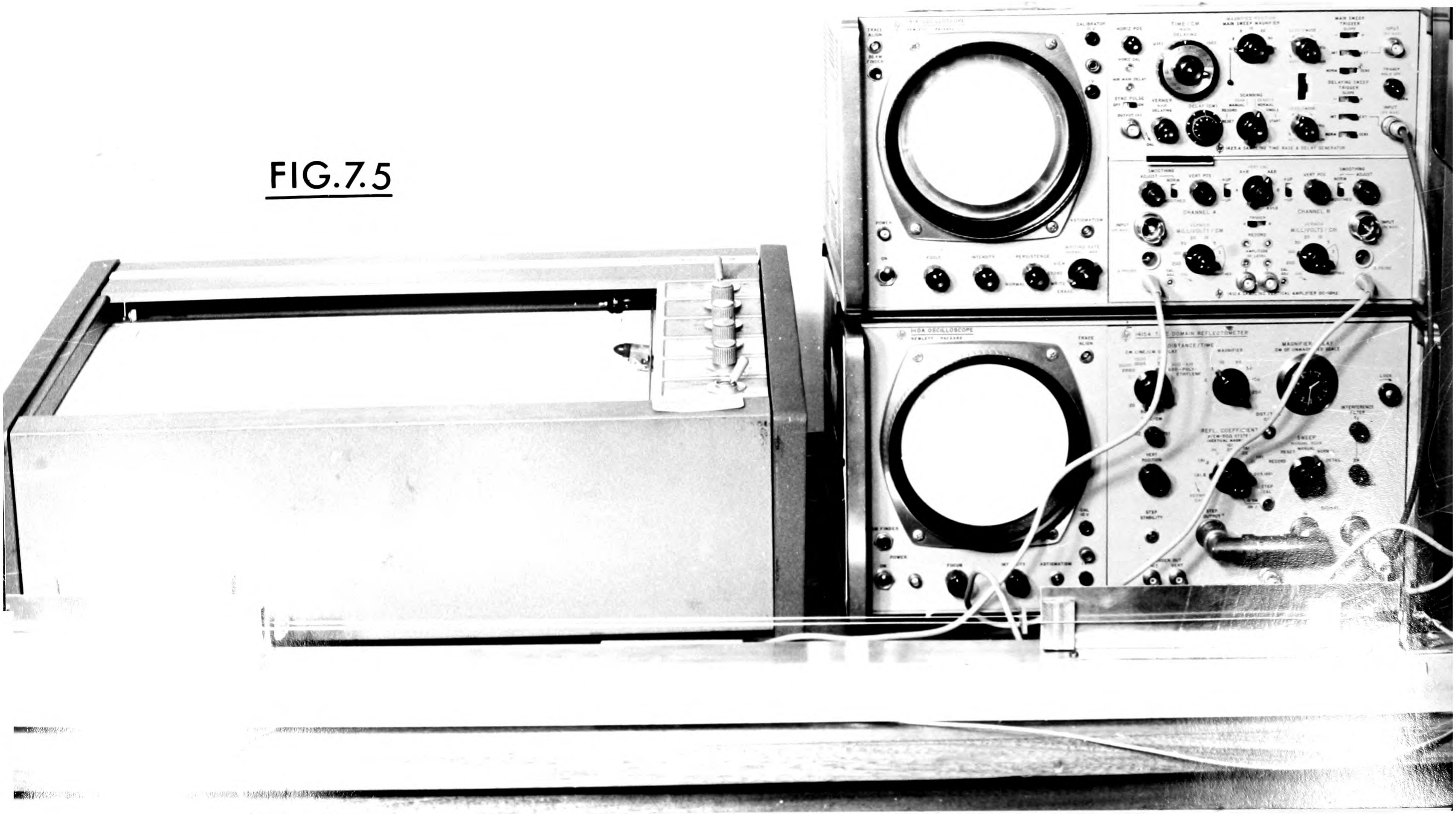
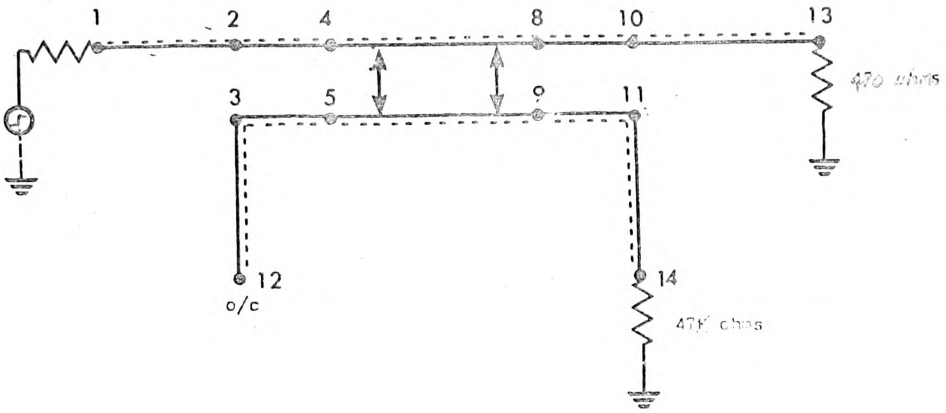

HP 1410A OSCILLOSCOPE
HEWLETT · PACKARD
1415A TIME DOMAIN REFLECTOMETER
CHANNEL A
CHANNEL B
MILLIVOLTS / CM

FIG.7.5



LINE DATA

node	to node	self surge imp. (ohms)	mut. surge imp. (ohms)	length. (m)	velocity factor
2	1	50.0	-	1.3	0.66
2	4	259.0	-	0.25	1.00
3	5	259.0	-	0.25	1.00
4	8	259.0	70.0	0.5	1.00
5	9	259.0	70.0	0.5	1.00
8	10	259.0	-	0.25	1.00
9	11	259.0	-	0.25	1.00
10	13	50.0	-	1.3	0.66
3	12	50.0	-	0.65	0.66
11	14	50.0	-	0.65	0.66

source impedance = 50 ohms.

FIG.7.6

line having two conductors above an earthed plane. The remainder of the network comprised co-axial cables and the excitation was provided by a time domain reflectometer output. A sampling oscilloscope with high impedance probes enabled voltage levels to be monitored where necessary. A general view of the experiment is shown in fig. 7.5 and fig. 7.6 describes the characteristics of the network in detail.

Comparisons between measured results in the laboratory and the results of a simulation using SUSAN are made in fig. 7.7. In general agreement both in the form of the transients and their amplitudes was very good with discrepancies amounting to no more than a few per cent. The system chosen for these tests was quite complex and it is believed that at the time of writing SUSAN is probably the only program available capable of handling it.

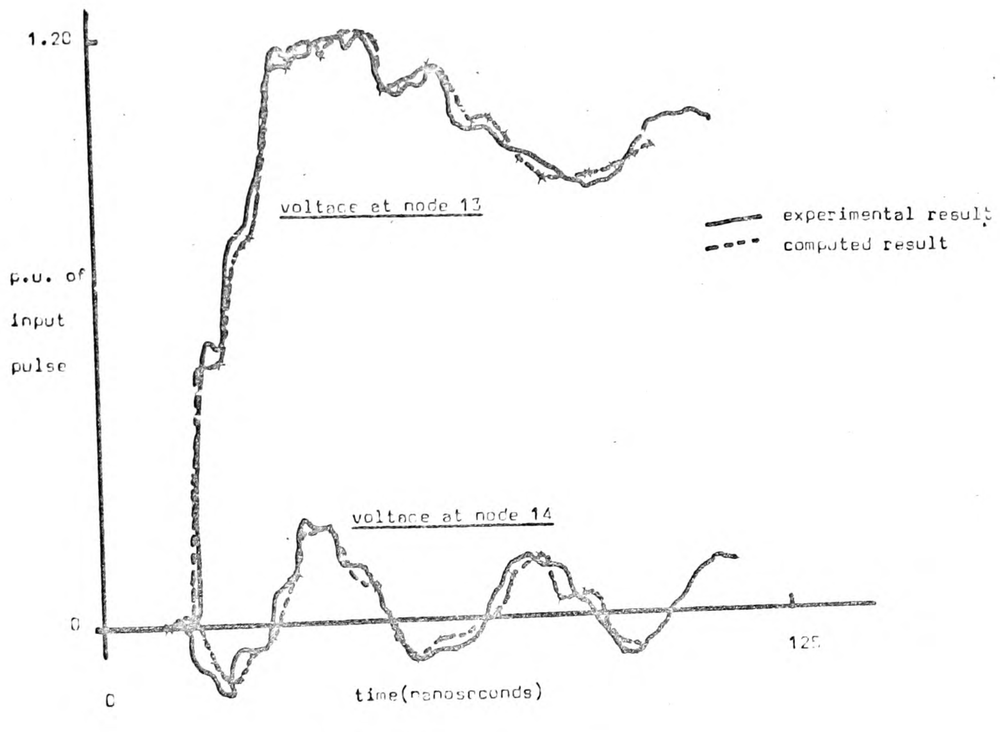

1.20
voltace at node 13
experimental result
computed result
p.u. of
input
pulse
voltace at node 14
0
125
0
time(nanoseconds)

FIG.7.7

8 CONCLUSIONS

8.1 SUSAN as an engineering tool

The stated intention of this work has been to prepare a digital computer program for transient calculation in large electrical networks. The extent to which the system which has resulted fits this requirement can only be judged by its users. It has been shown, however, that this program can produce meaningful and, if is hoped, useful results on a wide variety of networks covering a range of applications.

The size - in terms of core storage - and the power of the computer used for most of this work has necessarily been limited. These apparent disadvantages have been reflected in the size of the networks which have been actually analysed. Thus the term 'large' in the first sentence of the preceeding paragraph thus cannot be taken too literally, though very simple modifications to the programs - amounting to no more than increasing the sizes of various matrices and vectors together with a few smaller modifications - are all that are required to render the system capable of handling networks of thousands of nodes. The same ease of use which has been one of the objects of the exercise will be retained.

It might be argued that programs such as this are better developed on large computers, though this is not a view to which the writer would subscribe, since it is felt that extravagances of storage and huge computing power tend to result in the production of rather inefficient routines. The limitations imposed by a small computer, whilst making the programming effort more difficult, do prompt the programmer to seek more efficient, neater and more elegant solutions to the many problems with which he is confronted.

It is not suggested that SUSAN has reached anywhere near its ultimate in development. Transient programs of this kind have only been available for a very few years and work in this field has not been so intense as in other regions of network computation. Within the limits of the present 'state of the art', however, programs such as this based on the graphical method would seem to provide a practicable general purpose tool for the study of network transients. Greater accuracy can doubtless be achieved by other techniques which can take, for example, line losses into account more exactly. Before such programs can make a real contribution it is certain that more and better data is required about the performance of individual network components under transient conditions. In the power engineering field especially where transient calculations are becoming more important, simple measurements on new and existing plant like stray capacitances and surge impedances must be made in future.

8.2 Further work in this field

A number of avenues into which this work could be directed come to mind. The program as it stands can doubtless be applied to a number of situations where it may result in more of the 'approximations' of the type for the capacitive loading of lines as described in chapter 6. We have been more concerned with the development of a tool - the program - rather than its applications, and it is therefore relevant to consider some of the additional features which might be incorporated to make that tool more powerful.

8.2.1 Non-linear lines

The solution of problems in networks which incorporate non-linear lines - whether this non-linearity be due to variable dimensions in the line structure or to artificial lines whose elements are made voltage or current dependent - is an obvious field for investigation.

The direct application of the graphical method to this type of network is relatively straightforward in the case of lines containing lumped non-linear elements. A previous reference[29] has already indicated how such elements may be approximately handled. In the case of 'true' non-linear lines, however, the graphical method loses its essential simplicity since straight lines on the voltage/current plane become curves. Fortunately recent work by Giguere[51] indicates that it is reasonable to treat these non-linear lines as cascades of linear lines. It is thus theoretically possible for the system as it stands to deal with such lines though a more sophisticated treatment is clearly desirable.

8.2.2 Multi-phase analysis

The multiphase capability of SUSAN is presently restricted to two phases, though it has been pointed out that this is not a fundamental limitation of the program, only a practical consequence of limited storage in the computer. Obviously further work will be carried out to extend the program to handle more phases, but an early temptation to write two separate programs, one for single phase work and one for multiphase analysis will be ignored since great advantages obviously accrue from being able to include coupled and uncoupled line sections in the same network.

8.2.3 Combination with other methods

It would seem unlikely that the graphical method as presently interpreted can be combined with another process to form a 'hybrid' technique. It has been mentioned that this has been done with the basic lattice method where its combination with Fourier analysis leads to a powerful approach. It therefore seems that the graphical process will, at least for the moment, have to 'stand on its own feet'. It is, however, a most powerful tool in the armoury of the electrical engineer and a tool whose applications have only just started to be appreciated.

8.3 Range of Application of the Program

During the past few years a number of programs have been developed for the analysis of electrical networks. Many of these would claim to be 'general purpose' in nature, though the rather specialised requirements of power systems analysis imply that their main area of application lies in the light current field.

Possibly the best known of all the circuit analysis routines available is the I.B.M. Electronic Circuit Analysis Program (ECAP) which offers facilities for D.C. and A.C. steady state and transient analysis of a wide range of networks. Mention must also be made of the REDAC computer aided design packages which are now readily available to the circuit designer. The SUSAN system is essentially intended for time domain transient analysis, and it should therefore be compared with other commercial transient analysis routines. It might conceivably be argued that SUSAN could be employed for steady state analysis - by extending the run time until the transient portion of the response has subsided - but such a policy could hardly be regarded as being at all practicable.

Taking the REDAC system as a typical example, this offers an option (REDAP 16) for non-linear transient analysis. The non-linearities available cover such items as non-linear resistors, diode and transistor models and variable capacitances. The essential difference between REDAP 16 and SUSAN, however, is that the former program makes no provision for the inclusion of distributed constant transmission lines. Certainly lines can be simulated by 'pi' sections - an approximation which may be considered suitable for very short lines. In general, however, it must be argued that REDAP 16, and most if not all of the other commercial analysis routines are intended for lumped element circuit analysis only. SUSAN therefore fills a vacuum in providing a calculation system for those networks in which transmission lines are a dominant feature.

By considering lumped circuit elements to be composed of the lumped elements coupled together by very short transmission lines, it is, in theory, possible to use SUSAN for the analysis of 'conventional' circuits. In such a role it is then in direct competition with programs such as REDAP 16. SUSAN is not, however, intended for use in this manner when it would prove to be most inefficient and probably inaccurate - since the parameters of the coupling transmission lines would be arbitrarily determined - in comparison with the other programs mentioned.

8.4 Originality of the Work

Certain of the features of the program and the analyses contained within this work are considered to be original. They have not been incorporated into other transient analyses programs prepared by the writer, nor, as far as is known, in similar programs from other sources. The specific items are:-

(a) The method adopted for the storage and processing of network topological data. (see page 66)

(b) 'Past History' analysis. (see page 68)

(c) Automatic sectioning of transmission lines to obtain an optimal structure bearing in mind the quantity of fast storage area available. (see page 54)

(d) The facility to handle series lumped elements such as resistors and coupled coils between multi-line junctions. (see page 53)

(e) The analyses for transient computation in multi-phase systems using:-

(i) 'Modal' techniques and

(ii) The mutual surge impedance concept.

(f) The ability to analyse networks containing both mutually coupled line sections and uncoupled transmission lines within the same circuit.

(g) The program output options, especially the off-line graphical technique using a hybrid computer. (see page 64)

(h) The 'graph truncation' option permitting output data to be expressed in a condensed and thus more convenient form. (see page 64)

(i) The approximation for the response of transmission lines with capacitive terminations. (see page 79)

(j) The transient analysis of a circuit containing both transmission lines and mutually coupled inductors.

Additionally it may be pointed out that the overall level of sophistication of the SUSAN program - in terms of ease of use, freedom from topological restrictions etc. - is considerably higher than that of earlier programs prepared by the writer and others.

APPENDICES

Appendix 1 - Program listing

Appendix 2 - Node type coding information

Appendix 3 - System messages

Appendix 4 - Published papers relating to the work

(Note: Appendices 1 and 4 are contained in vol. 2)

APPENDIX 2

SURGE SYSTEM ANALYSIS PROGRAM - NODE TYPES

Node Type		
1 (simple node)	A = B = C = D = E = 0	
2 (voltage source)	A = 1 (step function) B = step amp. C = D = 0	volts x mult.
	A = 2 (ramp function) B = rate of rise of volts. C = D = 0	volts x mult. /microsec.
	A = 3 (sinusoid) B = peak voltage C = frequency D = initial phase angle	volts x mult Hz x 100 degrees
	A = 4 (double exponential) $v = B.(e^{-Ct} - e^{-Dt})$	B in volts x mult.
	A = 5 (pulse gen.) B = rate of rise/fall of voltage (note: pulse amplitude = 5v.) C = overall pulse width ('mark') D = 'space' E = source impedance in each case	volts x mult /microsec. microsecs. x mult. microsecs. x mult. Ohms x 100
3 (resistor to ground)	A = B = C = D = 0 E = resistor value	ohms x 100
4 (non-linear parallel with linear resistor termination)	resistor law is: $i = A.v^B + C$ D = 'flashover' level E = value of parallel linear resistor if present	ohms x 100
5 (short circuit to ground)	A = B = C = D = E = 0	
6 (series capacitor)	A = node number to which other end of capacitor attached B = value of capacitor C = D = E = 0	microfarads x 0.01

Node Type		
7 (series resistor)	A = node to which other end of resistor attached B = value of resistor (non-zero) C = D = E = 0	 ohms x 100
8 (coupled coils)	A = node to which other coil terminal connected B = value of inductance on this side C = value of inductance on other side D = coupling coefficient (+ for coils in same sense (starts together), - for coils in opposite sense) E = 0	 mH x 0.01 mH x 0.01
9 (mutually coupled transmission lines)	A = corresponding node at same end of coupled line group, but on other line. B = line self surge impedance C = line mutual surge impedance D) E) - zero if line sections both sides of node are coupled. If line sections on only one side of the node are coupled, then D & E hold numbers of adjacent nodes on the coupled side. Note: Restriction on numbering. The following numbering restriction must be adopted when type 9 nodes are used:- Two coupled lines contain type 9 nodes j & k as follows:-  a,b,c and d are immediately adjacent to j and k as shown. The numerical value of a must be less than b, <u>and</u> the value of c must be less than d. Line lengths aj, jb, ck and kd must be defined in the data input as equal to one time unit in the sectioned network.	 ohms ohms

APPENDIX 3

SUSAN - SYSTEM MESSAGES

Code No.	Message	Interpretation	Operator Action
S01	Node XXX missing from data set	Nodes must be specified sequentially from 1 onwards. A break in the sequence has been detected, XXX is the number of the missing node. Run abandoned, no recovery possible.	None Control returns to Supervisor
S02	Node XXX is not permitted	A node numbered zero or with too high a number has been detected. Numbering sequence must start with 1. Run abandoned, no recovery possible.	None Control returns to Supervisor
S03	Negative node number detected	A node having a negative number has been detected which is invalid. Run abandoned, no recovery possible.	Non Control returns to Supervisor.
S04	Connection to unspecified node	A connection has been specified as having terminating nodes one or both of which have numbers greater than the highest numbered node in the network. Run abandoned, no recovery possible.	None Control returns to Supervisor
S05	Connection made to itself	Both the terminating node numbers of a connection have been given the same number - i.e. the connection is made to and from the same node. Run abandoned, no recovery possible.	None Control returns to Supervisor
S06	Node XXX specified twice	Two nodes have been given the same number (XXX). Each node must have a different number. Run abandoned, no recovery possible.	None Control returns to Supervisor
S07	Type 2 node (XXX) possible data error	Extraneous and unnecessary data has been located in either the C or D parameter positions on the data card for node XXX. Spurious data is ignored, processing continues.	None

Code No.	Message	Interpretation	Operator Action
S08	Type 2 node (XXX) possible data error	Extraneous and unnecessary data has been located in either the C or D parameter positions on the data card for node XXX. This node has been specified as having a ramp function generator connected to it. Spurious data is ignored, processing continues.	None
S09	Type 3 node (XXX) possible data error	Extraneous data has been located in one of the A to D parameter positions of the data card for node XXX. Spurious data is ignored, processing continues.	None
S10	End of node data. No fatal errors detected.	All available node data has been read and the error checks have failed to determine any irregularity.	None
S11	No detectable errors in connection data	Error checks have failed to reveal any obvious errors in the connection data.	None
S12	Run abandoned due to errors	Errors have been detected in the network data causing this and subsequent runs to be abandoned. No recovery possible.	None Control returns to Supervisor.
S13	XX additional nodes added following sectioning	XX additional nodes have been added to the network in addition to those originally specified following the action of the sectioning routine. This figure does not include nodes added to simulate reactive devices.	None
S14	Sectioning tolerance increased to XX percent	The program attempted to section the system to the specified sectioning tolerance. It was found that too much storc was required and the tolerance was increased to XX percent.	None
S15	Study duration is XXXXXX.XXX microsecs	The figure here is the specified study duration in microseconds.	None

Code No.	Message	Interpretation	Operator Action
S16	Switch 1 on for section data Push Start	Details of the network topology after line sectioning may be optionally printed. To obtain this information switch 1 should be set on now. Program pauses for response.	Set switch 1 on or off Push program start.
S17	Switch 6 on if new run follows this. Push start.	SUSAN runs may be made sequentially without the need to re-establish the program between each. Data for the second and subsequent runs follows directly on the data for the first. Presence of a following run is indicated by switch 6 setting. Program pauses for operator response.	Set switch 6 on or off Push program start.
S18	Node modification made at time XXXXXX.XXX microsecs. New data is:-	A modification to the parameters of one of the nodes in the network has been made at time XXXXXX.XXX microsecs. The new data is echo checked and this message will appear each time such a modification is carried out.	None
S19	Processing will continue	This message appears following the detection of a possible error in the input data. Such an error will not cause the run to be abandoned.	None
S20	Specified sectioning tolerance is XXX per-cent.	The sectioning tolerance as specified in the input data is printed here for reference.	None
S21	Length increase made in line from XXX to YYY	A connection was specified as being so short in relation to other connections in the network that too much store was required. As a result the connection between XXX and YYY has been lengthened. It is recommended that switch 1 option should be used if this message appears. (See S16 above)	None

Code No.	Message	Interpretation	Operator Action
S22	Switch 2 on for graphical output, off for numerical output. Push start	The program output may be obtained either in conventional tubular form or in graphical form. The choice of output form is governed by the status of switch 2 at this time. Program pauses for operator response.	Set switch 2 on or off Push program start.
S23	Graphical output requested. Switch 3 on for auot-scaling, off for manual. Push start.	This message should only appear in response to switch 2 being on for graphical output (see S22). The graph may either have a max. and min. voltage ordinates supplied by the user, or these may be computed automatically by the program (auto-scaling). In the auto-scaled case the graph will exactly fit the page. Option is selected by switch 3 and the program pauses for operator response.	Set switch 3 on or off Push program start.
S24	Specify scale factors (VMIN,VMAX) BXXXX.XXXBXXXX.XXX	This message should only appear in response to switch 3 being off for manual graph ordinate selection (S23). The max. and min. voltage scale ordinates should be types in to the format shown (VMIN first, B is blank or -ve. sign). Program selects keyboard input and pauses. Ordinates specified apply to all graphs in the run.	Type in VMIN and VMAX to format specified in message ('B' is blank or -ve. sign) Push End of Field (EOF) to continue processing.
S25	No shunt reactive elements in network	The routine has been unable to detect the presence of any shunt reactive elements in the network data. This fact is printed for reference. Processing continues.	None
S26	End of shunt reactor data	The program has detected data on shunt reactive elements in the network. This message marks the end of that data. Processing continues.	None

Code No.	Message	Interpretation	Operator Action
S27	Switch 4 on for graph truncation. Push Start.	This message will occur before each graph if the graphical output option has been selected. (see S22,S23). If switch 4 is placed on then parts of the graph during which there is no change in voltage for long periods will not be printed and printing will only take place in regions of interest. Program pauses for operator response.	et switch 4 on or off. Push program start.
S28	Warning - the following graph has a non-linear time axis.	This message appears only after switch 4 has been placed on for graph truncation (S27). In that event portions of the graph may have been omitted a and consequently the time axis will appear non-linear. Interpretation of the graph should be done with care.	None
S29	Switch 5 can terminate run at any time	The run may be terminated at any time during the transient computation phase by turning switch 5 on. The results obtained prior to this action will be printed or graphed in the normal way. This facility is intended for occasional use in the event that the run must be cut short.	Set switch 5 on when premature stopping of run required.
S30	Run terminated manually	This message is printed if premature run termination was carried out by the use of switch 5 (see S29).	None
S31	Switch 7 on if results to be stored in data bank. Push start.	The data produced about the transient behavious of the voltage at any of the output nodes may be stored on the files associated with the system if required for future use - e.g. for plotting.	Set switch 7 on or off. Push start.

Code No.	Message	Interpretation	Operator Action
S32	Type result node no. (XXX) and file no. (YYY). Format is XXXYYY	The program expects the user to type data on node number whose results are to be stored and file number $(01 \text{ to } 10)_{dec.}$ in which these results are to be placed. Message only produced if switch 7 was on. (see S31)	Type data. Terminate with End of Field (EOF) key.
S33	Results not computed for node XXX - Re-type data.	Error in typing data for result storage. (see S31 and S32). Correct data must be entered.	Repeat action for S32.
S34	Node XXX results stored on file YYY.	Storage operation as in S31 and S32 above has been completed satisfactorily.	None.
S35	Capacitor connected to itself. Node XXX	A series capacitor has been specified as connected to itself. - i.e. the 'A' field in the additional data associated with the type 6 node is equal to the node number. Node is XXX, no recovery possible.	None. Control returns to Supervisor.
S36	Capacitor of zero magnitude detected	A series capacitor of zero magnitude has been detected in the network. No recovery is possible	None. Control returns to Supervisor.
S37	Too many series capacitors	More than four separate series capacitors have been detected in the network. No recovery possible.	None. Control returns to Supervisor.
S38	Capacitor between nodes XXX and YYY has too many connections.	The series capacitor shown connected between nodes XXX and YYY has more than four connections in total at its two terminals. Run abandoned, no recovery possible.	None. Control returns to Supervisor.

Code No.	Message	Interpretation	Operator Action
S39	Resistor connected to itself Node XXX	A series resistor has been shown connected to itself - i.e. the 'A' field of the additional data associated with the type 7 node is equal to the node number. Run abandoned, no recovery possible.	None Control returns to Supervisor.
S40	Too many series resistors	More than four separate series resistors have been detected in the network. Run abandoned, no recovery possible.	None. Control returns to Supervisor.
S41	Resistor between nodes XXX and YYY has too many connections	The series resistor connected between nodes XXX and YYY has more than four connections in total to its two terminals. Run abandoned, no recovery possible.	None. Control returns to Supervisor.
S42	Too many coupled coils	More than one coupled coil system has been detected in the network. Run abandoned, no recovery possible.	None. Control returns to Supervisor.
S43	Coupled coil data error - zero inductance	An error in coupled coil data has been detected whereby one or other of the self inductances is zero or has a negative sign. Run abandoned, no recovery possible.	None. Control returns to Supervisor.
S44	Coupled coils in parallel - Node XXX	Coupled coils have been shown connected to themselves at node XXX - i.e. the 'A' field in the data associated with the type 8 node is equal to the node number. Run abandoned, no recovery possible.	None. Control returns to Supervisor.
S45	Coupled coil data error - coefficient value	An error in coupled coil data has been detected whereby the coupling coefficient lies outside the range +/- 1. Run abandoned, no recovery possible.	None. Control returns to Supervisor.

Code No.	Message	Interpretation	Operator Action
S46	Coupled coils between nodes XXX and YYY have too many connections	The number of connections, including shunt reactive elements, to the terminal nodes of the set of coupled coils between nodes XXX and YYY exceeds four. Run abandoned, no recovery possible.	None. Control returns to Supervisor.
S47	Timed node modification error. Mod deleted.	An attempt has been made to modify the data at a type 6, 7 or 8 node. This is not permitted. The modification is ignored and processing continues.	None.

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